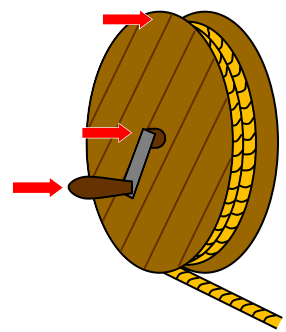
Rotation

There are a number of different ways of quantifying rotation. We have multiple variables for the velocity of a spinning object and the acceleration of a spinning object. It's important to know what each of these variables means, when each should be used, and what units each one is measured in.

The primary difference between the variables is whether they take the radius into account. This distinction is especially useful because it will help you decide which alphabet the variable name comes from. Just as in trigonometry, angles and angular quantities are expressed with Greek letters. Distances, like the radius, are expressed with letters from the Latin (English) alphabet.

ANGULAR AND LINEAR VELOCITY



What do we mean by taking the radius into account? Consider the winch in the diagram at the right. Three locations are marked on it: the end of the handle, the axle, and a point on the edge of the reel. Imagine turning the handle to wind the rope onto the reel.

In one sense, the entire device has the same speed: it spins at the same rate no matter which point we're discussing. All three points are at different radii, but they all take the same length of time to make a complete rotation. This is the **angular velocity**. It's a measure of how many revolutions (or what total angle) an object travels in a given unit of time. The variable for angular velocity is **omega**: $\boldsymbol{\omega}$. (This letter looks like a w, but it's always written with curves, never with four straight lines.) The proper unit for angular velocity is $^{rad}/_{s}$.

In another sense, some parts of the winch are travelling faster than others. The axle never moves, and without motion there's no velocity. The point on the edge of the reel moves a long distance, the full circumference of the winch, with every rotation. The handle moves but not as far as the edge point. Here the radius makes a difference: the greater the radius, the more distance covered in the same time, so the higher the velocity. This velocity is the more traditional way of describing velocity. For a spinning object, it's called **linear velocity** or **tangential velocity** since its direction is tangential to the circular path. The variable is v (or sometimes $\mathbf{v_t}$), and it's measured in $^{m}/_{s}$. Interpretations of linear velocity for a spinning object include:

- for a wheel, the linear velocity at the circumference is equal to the distance the wheel travels along the ground
- for the winch in the picture, the linear velocity at the distance from the axle to the rope is equal to the amount of rope that comes onto (or off of) the reel

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ANGULAR, TANGENTIAL AND RADIAL ACCELERATION

There is an additional interpretation of acceleration where a spinning object is concerned.

It should not surprise you that **angular acceleration** is defined as the rate at which angular velocity is changing over time. The variable for angular acceleration is **alpha**, **α**, which is the Greek version of the letter "a".

There's also **tangential acceleration**, which is the rate at which tangential velocity is changing over time. It's measured in m/s^2, and its variable is $\mathbf{a_t}$. The reason for the subscript "$_t$" is because the third description of acceleration is also measured in meters per second squared, and it also describes the rate of change of linear velocity. For an object spinning at a constant angular acceleration, the kinematics equations you already know involving t, Δx, v and a will apply, with v_t for velocity and a_t for acceleration.

The **radial acceleration** of a rotating object uses the vector definition of acceleration: acceleration is any instance of speeding up, slowing down or changing direction, or any combination of these. Even for an object that isn't spinning any faster or any slower (i.e., the object is experiencing uniform circular motion, and there is no angular acceleration) the velocity vector for a point on the object changes — it changes direction even if its magnitude is the same. This means that the actual acceleration vector for such an object points at the centre of rotation for the object. Its units are m/s^2 and the variable we use is $\mathbf{a_r}$. For objects that are experiencing uniform circular motion (i.e. there is no angular acceleration) this radial acceleration is also called **centripetal acceleration**. For objects experiencing non-uniform circular motion, and angular acceleration, then the overall acceleration of a point on the object is the vector sum of a_t and a_r.

FORMULAS

The formulas that we use in calculations involving angular velocities and angular accelerations look very similar to the kinematics equations we use for linear motion. The main difference is that the variables have been replaced: "v" has been replaced by ω and "a" has been replaced by α. "Δs" or any other way of writing displacement is replaced by Δθ, which is called the **angular displacement**, and is measured in radians.

LINEAR	ANGULAR
$v = \dfrac{\Delta s}{\Delta t}$	$\omega = \dfrac{\Delta\theta}{\Delta t}$
$\Delta s = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$	$\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha(\Delta t)^2$
$a = \dfrac{v_f - v_i}{\Delta t}$	$\alpha = \dfrac{\omega_f - \omega_i}{\Delta t}$
$(v_f)^2 = (v_i)^2 + 2a\Delta s$	$(\omega_f)^2 = (\omega_i)^2 + 2\alpha\cdot\Delta\theta$

Radians describe angles in terms of arc length on a circle with a radius of 1 (called a unit circle). An angle of 1 radian covers a distance around the circumference of the unit circle equal to the radius. This fact makes working with conversions between linear and angular measures easy. The angular unit is the linear unit divided by the radius:

$$\theta = \frac{s}{r} \qquad \omega = \frac{v}{r} \qquad \alpha = \frac{a_t}{r}$$

To calculate centripetal or radial acceleration:

$$a_r = \frac{v^2}{r} = \omega^2 r\text{, in the direction of the axis of rotation}$$

The magnitude of the net acceleration of an object in non-uniform circular motion is:

$$a = \sqrt{a_r^2 + a_t^2}$$

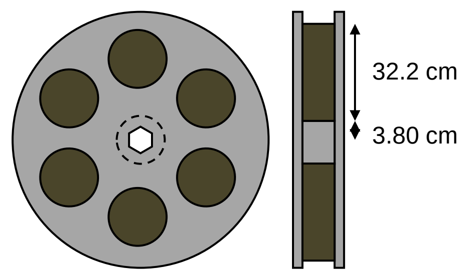
You will not be asked for the direction of net acceleration, since it's constantly changing.

EXERCISES

A. 1) Under what circumstances can you use the first of the equations in the “Linear” column on page 2, and under what circumstances can you use the other three?

2) Under what circumstances can you use the first of the equations in the “Angular” column on page 2, and under what circumstances can you use the other three?

B. A gear in an old film projector moves a film past the lens at a steady rate. Film displays 24 frames per second, and each frame is 1.86 cm long in the direction of the film. The film is currently being fed off a reel with the radius of 3.80 cm, and the film is 32.2 cm thick on the reel.



1) What is the linear velocity of the film passing by the lens?

2) What is the current tangential velocity of the film reel?

3) What is the current angular velocity of the film reel?

4) What will be the reel's final angular velocity when the film is about to run out?

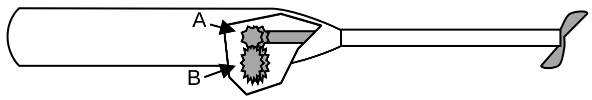
5) What will be the final linear velocity of the film reel at that time?

6) Does the film reel experience angular acceleration? Tangential acceleration? Radial acceleration?

7) If the film is 1 hour long, what is the average tangential acceleration of the reel over the length of the film? What is the average angular acceleration in that time?

8) If you compare your answers to Question 7, you'll find that the formula $\alpha = \frac{a_t}{r}$ does *not* apply. Why is this?

C. Inside an electric hand mixer, there is a system of small gears. One gear (A) drives the mixer's large blades because the blades are connected to an extension of the gear's axle. There is another slightly larger gear (B) meshed with gear A as well.



1) Do gear A and the mixer blades have the same tangential velocity when they spin? Do they have the same angular velocity?

2) Do gears A and B have the same tangential velocity when they spin? Do they have the same angular velocity?

D. A man drives his motorcycle out of his straight driveway so that he's going 25.0 $^{km}/_{h}$ after travelling 49.0 m, the full length of the driveway. A piece of gravel from the driveway is lodged in the tire tread at the edge of the tire. The tires are 86.0 cm in diameter.

1) What acceleration did the biker achieve?

2) What is the linear velocity of the piece of gravel after 49 m?

3) What is the angular velocity of the tires at the end of the driveway?

4) What radial acceleration is the piece of gravel experiencing when the biker gets to the end of the driveway?

5) What is the magnitude of the net acceleration that the piece of gravel is experiencing?

SOLUTIONS

A: (1) You use the first equation when the particle experiences no (tangential) acceleration, and the remaining three when there is constant acceleration. (2) You use the first equation when the particle experiences no angular acceleration, and the remaining three when there is constant angular acceleration.

B: (1) 0.446 $^{m}/_{s}$ (2) 0.446 $^{m}/_{s}$ (3) 1.24 $^{rad}/_{s}$ (4) 11.7 $^{rad}/_{s}$ (5) 0.446 $^{m}/_{s}$
(6) It experiences angular and radial acceleration, but not tangential.
(7) $a_t = 0$; $\alpha = 2.92 \times 10^{-3}$ $^{rad}/_{s^2}$ (8) The film reel does not have a constant radius, so there's no single value to plug into r. (It's the fact that the radius decreases that causes the angular acceleration. If the film could be looped back onto the same reel, like a conveyor belt, then both a_t and α would equal 0, and the equation would hold.)

C: (1) Because both objects are coaxial (like the winch and its handle) they have the same angular velocity. They're different sizes, so they do not have the same tangential velocity. (2) Because the two gears' teeth go by at the same rate where they are meshed, the gears have the same tangential velocity. If a_t is the same, but the gears' radii are different, they have different angular velocities; gear A's α is greater.

D: (1) 0.492 $^{m}/_{s^2}$ (2) 6.94 $^{m}/_{s}$ (3) 16.1 $^{rad}/_{s}$ (4) 112 $^{m}/_{s^2}$ (5) 112 $^{m}/_{s^2}$