# Differentiation & Integration Formulas

## DIFFERENTIATION FORMULAS

$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$

$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$

$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$

$\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$

$\frac{d}{dx}(\log_a u) = \frac{1}{u} \log_a e \frac{du}{dx}$

$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$

$\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$

$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

## INTEGRATION FORMULAS

Note: a, b and c are constants; k is the integration constant.

$\int a\, dx = ax + k$

$\int ax^b\, dx = \frac{a}{b+1} x^{b+1} + k,\ b \neq -1$

$\int \frac{a}{bx+c}\, dx = \frac{a}{b} \ln(bx + c) + k$

$\int ae^{bx+c}\, dx = \frac{a}{b} e^{bx+c} + k$

$\int a \sin(bx + c)\, dx = -\frac{a}{b} \cos(bx + c) + k$

$\int a \cos(bx + c)\, dx = \frac{a}{b} \sin(bx + c) + k$

$\int \sec^2 x\, dx = \tan x + k$

$\int \csc^2 x\, dx = -\cot x + k$

$\int \sec x \tan x\, dx = \sec x + k$

$\int \csc x \cot x\, dx = -\csc x + k$

$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + k$ **

$\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4} + k$ **

$\int \tan^2 x\, dx = \tan x - x + k$

## ** POWER-REDUCING FORMULAS

$\cos^2 x = \frac{1 + \cos 2x}{x}$

$\sin^2 x = \frac{1 - \cos 2x}{x}$

## SPECIAL LIMITS

$\lim_{x \to 0} \frac{\sin x}{x} = 0$

$\lim_{n \to \infty} (1 + \frac{x}{n})^n \stackrel{def}{=} e^x$

## L'HOSPITAL'S RULE

If you are asked to take the limit of a rational function $\lim_{x \to a} \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are differentiable, but the limit comes to $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$, assuming the second limit exists and $g'(x) \neq 0$.

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## INTEGRATION BY PARTS

Integration by parts is a way of using the Product Rule in reverse. The formula for integration by parts is:

$$\int u \, dv = u \cdot v - \int v \, du$$

Wikipedia (*http://en.wikipedia.org/wiki/Integration_by_parts*) suggests the following order for choosing which part of the integral to integrate and which to differentiate:

u
**L**ogarithmic functions
**I**nverse trigonometric functions
**A**lgebraic functions (such as $x^2$)
**T**rigonometric functions
**E**xponential functions
dv

Choose the part that is higher on the list for u, and the part that is lower for dv. This is a rule of thumb — it is a suggestion for what is best, but it doesn't always work perfectly.

## AREA UNDER A CURVE

The area between a curve $f(x)$ and the x-axis from x = m to x = n is:

$$A = \int_m^n f(x) \, dx$$

If a curve goes below the x-axis, the area in that section is *subtracted* from the total area.

It is possible to split integrals so that "negative area" is interpreted as positive. If, on the interval [m, n] containing p, $f(x) > 0$ over [m, p) and $f(x) < 0$ over (p, n], then

$$A = \int_m^p f(x) \, dx - \int_p^n f(x) \, dx$$

## VOLUME OF A SOLID OF REVOLUTION

If the region under the graph of $f(x)$, from x = m to x = n is rotated around the y-axis, then the volume swept out by the curve is:

$$V = \int_m^n 2\pi x \, f(x) \, dx$$

If the curve is rotated around the x-axis instead, the volume is:

$$V = \int_m^n A(x) \, dx = \int_m^n \pi \, [f(x)]^2 \, dx$$

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