Related Rates Problems

EXERCISES

1) A circular ripple is spreading out over a pond. The radius r is increasing at a rate of ½ m/s at the moment when r = 5 m. How fast is the disturbed area increasing at that moment?

2) Two variable quantities Q and R are related by the equations:
$$Q^3 + R^3 = 9$$
What is the rate of change $\frac{dQ}{dt}$ at the moment when Q = 2, if $\frac{dR}{dt}$ = 3 at that moment?

3) A ladder 26 m long leans against a vertical wall. The foot of the ladder slips. If the lower end is moving away from the wall at the rate of 3 m/s, how fast would someone standing at the top of the ladder be moving when the foot is 10 m away from the wall?

4) A rope 32 m long is attached to a weight and passed over a pulley 16 m above the ground. The other end of the rope is pulled away horizontally along the ground at the rate of 3 m/s. At what rate does the weight rise at the instant that it's 4 m above the ground?

5) A woman 1.8 m tall walks away from a streetlight that is 4.5 m high at a rate of 1.2 m/s.
    a) How fast is the tip of her shadow moving when she is 9 m from the pole?
    b) How fast is her shadow lengthening?

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6) Two cars start from the same parking lot at the same time. One travels north at 40 $^{km}\!/_h$, and the other travels east at 70 $^{km}\!/_h$. How fast is the distance between them increasing after one hour?

7) A tank in the shape of an inverted cone with an equilateral cross-section is filled with water at the constant rate of 10 $^L\!/_{min}$. How fast is the water rising when the water level in the tank is 50 cm? [*Hint: 1 L = 1 dm³ = 1000 cm³*]

8) An Erlenmeyer flask has a conical "body", vertex *up*. The radius at the bottom is 10 cm, and the height of the cone would be 10 cm also. Water is poured into the flask at a rate of 5 $^{mL}\!/_s$. How fast is the water level rising when the water in the flask is 5 cm deep?

9) The **angle of repose** for a granular substance is the angle that the side of a pile of that substance can make with the ground. If the angle exceeds the angle of repose, it spills down and the pile takes up more space on the ground.

　　Sand falls on a conical pile at a rate of 30 $^{m³}\!/_{min}$. The angle of repose for this grade of sand is 50°. How fast is the space on the ground the pile is occupying increasing when the pile is 7 m tall?

SOLUTIONS
A. (1) 5π $^{m²}\!/_s$　(2) $-\frac{3}{4}$　(3) 1.25 $^{m}\!/_s$ downward　(4) 1.8 $^{m}\!/_s$　(5)a) 2 $^{m}\!/_s$　(b) 0.8 $^{m}\!/_s$

　　(6) $\approx$ 81 $^{km}\!/_h$　(7) $^{12}\!/_\pi$ $^{cm}\!/_{min} \approx$ 3.8 $^{cm}\!/_{min}$　(8) $\frac{1}{5\pi}$ $^{cm}\!/_s \approx$ 0.6 $^{cm}\!/_{min}$　(9) $\frac{60}{7}$ $^{m²}\!/_{min} \approx$ 8.571 $^{m²}\!/_{min}$