Differentiation Rules

CONSTANT RULE

$\frac{d}{dx} n = 0$, where $n \in \mathbb{R}$

The derivative of a constant is zero.

POWER RULE

$\frac{d}{dx} x^n = nx^{(n-1)}$, where $n \in \mathbb{R}$, $n \neq 0$

The derivative of a power of x is the old exponent times x raised to the old power minus one

COEFFICIENT RULE

$\frac{d}{dx} n \cdot f(x) = n \cdot \frac{d}{dx} f(x)$

The coefficient of an expression doesn't affect the derivative of that expression.
expressions.

SUM & DIFFERENCE RULES

$\frac{d}{dx} (f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

The derivative of a sum (or difference) of expressions is the sum (or difference) of the derivatives of the individual

BASIC DERIVATIVES

$\frac{d}{dx} x = 1$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \ln x = \frac{1}{x}$, $x > 0$

Example 1: Use one of the rules to find the derivative: a) x^4 b) $4x$ c) $\sin x + \cos x$.

Solution: a) Power Rule: $\frac{d}{dx} x^4 = 4x^3$
b) Coefficient Rule: $\frac{d}{dx} 4x = 4 \cdot 1 = 4$
c) Sum Rule: $\frac{d}{dx} (\sin x + \cos x) = \frac{d}{dx} \sin x + \frac{d}{dx} \cos x = \cos x + (-\sin x)$

PRODUCT RULE

$\frac{d}{dx} (f(x) \cdot g(x)) = f'(x)g(x) + g'(x)f(x)$

QUOTIENT RULE

$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

You will sometimes see the Product Rule written slightly differently, but remembering that the derivative of the first part comes first (though the order doesn't matter in the end) will help you to remember the same pattern in the Quotient Rule (where the order does matter).

Example 2: Use the Product Rule to find the derivative of $x \cdot \ln x$.

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Solution: $\frac{d}{dx}\, x \cdot \ln x = (1 \cdot \ln x) + (x \cdot \frac{1}{x}) = \ln x + 1$

OTHER DERIVATIVES

$\frac{d}{dx} \csc x = -\csc x \cdot \cot x$ $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$ $\frac{d}{dx} a^x = a^x \ln a$, $x > 0$

$\frac{d}{dx} \sec x = \sec x \cdot \tan x$ $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1 - x^2}}$ $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$, $x > 0$

$\frac{d}{dx} \cot x = -\csc^2 x$ $\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$

EXERCISES

Note: You should be able to do these without referring to the previous page!

A. Find the derivative:

1) $\ln x - x$
2) $\frac{1}{x}$
3) x^{17}
4) $5 \cos x$
5) e
6) $\sqrt[3]{x}$

B. Use the definitions of the following functions and the Quotient Rule to prove the list of derivatives given at the top of this page.

1) csc x 2) sec x 3) cot x

C. Find the derivative. Do not simplify the results:

1) $6x^3 + 5x^2 - 7x + 2$
2) $\frac{3}{57}x^{19} - \frac{4}{45}x^{15} + \frac{5}{165}x^{11}$
3) $(x - 3)^2$
4) $\sec x \cdot \ln x$
5) $(x^3 - 8x^2 + x)(\log x)$
6) $\frac{3x^4 + 17x^3 - 14x^2 - 73x + 63}{x^2 + 4x - 9}$
7) $\frac{\cos x}{\cos^{-1} x}$
8) $3x \cdot x^3 \cdot 3^x$ [*Hint: there's a trick.*]

SOLUTIONS

A. (1) $\frac{1}{x} - 1$ (2) $-\frac{1}{x^2}$ (3) $17x^{16}$ (4) $-5 \sin x$ (5) 0 (6) $\frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} = \frac{\sqrt[3]{x}}{3x}$

B. (1) $\frac{-\cos x}{\sin^2 x} = -\csc x \cot x$ (2) $\frac{\sin x}{\cos^2 x} = \sec x \tan x$ (3) $\frac{-\sec^2 x}{\tan^2 x} = \frac{-1/\cos^2 x}{\sin^2 x/\cos^2 x} = \csc^2 x$

C. (1) $18x^2 + 10x - 7$ (2) $x^{18} - \frac{4}{3}x^{14} + \frac{1}{3}x^{10}$ (3) expand first; $2x - 6$

(4) $\sec x \tan x \ln x + \frac{\sec x}{x}$ (5) $(3x^2 - 16x + 1)(\log x) + \frac{x^2 - 8x + 1}{\ln 10}$

(6) $\frac{(12x^3 + 51x^2 - 28x - 73)(x^2 + 4x - 9) - (3x^4 + 17x^3 - 14x^2 - 73x + 63)(2x + 4)}{(x^2 + 4x - 9)^2}$, or $6x + 5$, if you divided first.

(7) $(-\sin x \cos^{-1} x + \frac{\cos x}{\sqrt{1 - x^2}})(\cos^{-1} x)^{-2}$ (8) $x^4 \cdot 3^{x+1} \cdot \ln 3 + 4x^3 \cdot 3^{x+1}$

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