VANCOUVER COMMUNITY COLLEGE

Calculus Review Sheet

DEFINITION OF A DERIVATIVE

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative represents the slope of a function, or the **rate of change** in the value of a function.

LIMIT PROPERTIES

$$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{provided} \lim_{x \to a} g(x) \neq 0$$

CONTINUITY

[1] A function $f(x)$ is **continuous** at a point x = a if and only if:

$$\lim_{x \to a} f(x) = f(a)$$

A function is continuous on an interval if it is continuous at every point in the interval.

[2] The following functions are continuous everywhere in their domains:

exponential functions [$f(x) = 2^x$]
logarithmic functions [$f(x) = \log_5 x$; $f(x) = \ln 2x$]
polynomials [$f(x) = x^5 - 4x^3 + \frac{1}{5}x^2 + 17$]
rational functions [$f(x) = \frac{x^2+7}{x+5}$]
root functions [$f(x) = \sqrt[3]{x^2}$]
trig functions and inverse trig functions [$f(x) = \sin x$; $f(x) = \cos^{-1} x$]

[3] If two functions f and g are continuous at a point c, then the following are continuous at c as well (k is a constant):

$f \pm g$ $\quad$ kf $\quad$ $f \cdot g$ $\quad$ $\frac{f}{g}$* $\quad$ $f \circ g$

* provided $g(c) \neq 0$

INTERMEDIATE VALUE THEOREM

If $f(x)$ is a continuous function over an interval [a, c],
$f(a) \neq f(c)$, and
B is a number between $f(a)$ and $f(c)$
then there must be a number b in [a, c] where $f(b) = B$

Authored by Gordon Wong

DIFFERENTIATION FORMULAS

$f(x) = k$	$f'(x) = 0$	(where k is a constant)
$f(x) = kx$	$f'(x) = k$	
$f(x) = g(x) \pm h(x)$	$f'(x) = g'(x) \pm h'(x)$	[Sum/Difference Rule]
$f(x) = g(x) \cdot h(x)$	$f'(x) = g(x) \cdot h'(x) + g'(x) \cdot h(x)$	[Product Rule]
$f(x) = \dfrac{g(x)}{h(x)}$	$f'(x) = \dfrac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{(h(x))^2}$	[Quotient Rule]
$f(x) = kx^n$	$f'(x) = knx^{n-1}$	[Power Rule]
$f(x) = g(h(x))$	$f'(x) = g'(h(x)) \cdot h'(x)$	[Chain Rule]

DIFFERENTIABILITY

[1] A function is **differentiable** at a point c if $f'(c)$ exists. A function is differentiable over an interval if it is differentiable at every point in the interval.

[2] *If* a function is differentiable over an interval, *then* it is continuous over the interval, but the reverse may not be true. (Just because a function is continuous, this does not mean it must be differentiable.)

CURVE SKETCHING

[1] A **critical point** is found on the curve at any x where $f'(x) = 0$.

[2] The curve is **increasing** on any interval where $f'(x) > 0$.
The curve is **decreasing** on any interval where $f'(x) < 0$.
The endpoints of these intervals can only be at $\pm\infty$, critical points, or discontinuities.

[3] An **inflection point** is found on the curve at any x where $f''(x) = 0$.

[4] The curve is **concave up** on any interval where $f''(x) > 0$.
The curve is **concave down** on any interval where $f''(x) < 0$.
The endpoints of these intervals can only be at $\pm\infty$, inflection points, or discontinuities.

The table below summarizes this information. For each feature in the body of the table, the rows tell you which graph (f, f' or f'') has the feature, and the columns indicate what the first or second derivative of $f(x)$ must be (positive, negative or zero) at those features. For example, to read the table, we would say that the graph of $f(x)$ is concave up when $f''(x)$ is positive. Another example is the graph of $f'(x)$ has a critical point when $f''(x)$ is zero.

THE GRAPH OF…	WHEN $f''(x)$ IS…			WHEN $f'(x)$ IS…		
	0	+	−	0	+	−
$f''(x)$	x-intercept					
$f'(x)$	critical point	increasing	decreasing	x-intercept		
$f(x)$	inflection point	concave up	concave down	critical point	increasing	decreasing