Trigonometric Functions of Angles

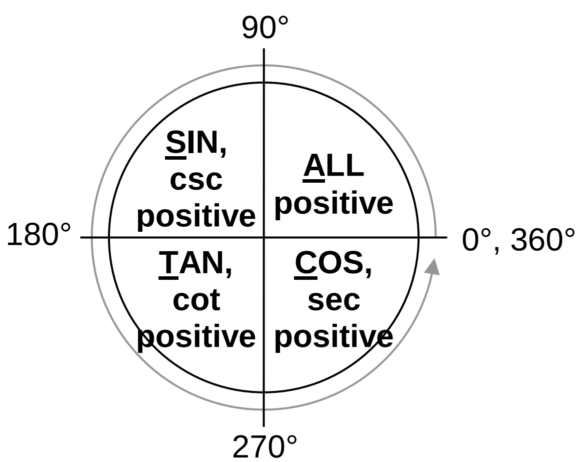
This worksheet tells you how to find trigonometric functions of given angles. For angles which lie on the x- or y-axis, such as 90°, the trigonometric function values can easily be obtained from the unit circle. For angles which do not lie on an axis, follow these steps:

[1] **Find the reference angle, θ_R, of the given angle θ.** The reference angle is the positive acute angle formed by the terminal ray of θ and the x-axis. If θ is greater than 360°, or is negative, it may be necessary to add or subtract multiples of 360° to get an angle between 0° and 360°. To summarize:

Quadrant I:	If	$0° < \theta < 90°$	then $\theta_R = \theta$.
Quadrant II:	If	$90° < \theta < 180°$	then $\theta_R = 180° - \theta$.
Quadrant III:	If	$180° < \theta < 270°$	then $\theta_R = \theta - 180°$.
Quadrant IV:	If	$270° < \theta < 360°$	then $\theta_R = 360° - \theta$.

[2] **Determine the trigonometric value of the reference angle.** If θ_R is 30°, 45° or 60°, then an exact answer is best. For other angles, use your calculator or trig tables to get the value.

[3] **Determine the sign of the trigonometric value of θ.** This chart will help you decide whether your answer is positive or negative:



We start counting angles from the positive x-axis and going counterclockwise. Because angles above 90° have at least one negative component, some of the trigonometric ratios are negative in Quadrants II, III and IV. To remember which ones are positive in each quadrant, use the mnemonic “All Students Take Calculus”. “All” stands for all six ratios. The initials of “Students”, “Take” and “Calculus” stand for sine, tangent and cosine. (Their inverses are positive in the same places as well, of course.)

Example 1: Find the exact value of cos 300°.

Solution: 300° is in Quadrant IV (between 270° and 360°). 360° − 300° gives a reference angle of 60°. The value of cos 60° is ½. Since cosines are positive in Quadrant IV, cos 300° = ½ as well.

Authored by Gordon Wong

Example 2: Find the exact value of cot 495°.

Solution: First we adjust 495° so that it is in the range from 0° to 360°. We do this by subtracting 360° (since, when going around the unit circle from the diagram, going past 360° means repeating the small angles). 495° − 360° = 135°. This angle is in Quadrant II, then. We calculate θ_R: 180° − 135° = 45°.

$$\cot 45^\circ = \frac{1}{\tan 45^\circ} = \frac{1}{1} = 1$$

Only sine (and cosecant) are positive in Quadrant II, therefore:

$$\cot 495^\circ = -\cot 45^\circ = -1$$

EXERCISES

A. Determine which quadrant θ is in based on the following information. (Note that the description of θ might not apply to every angle in a given quadrant):

1) sin θ is negative; cos θ is positive
2) tan θ is positive; sec θ is negative
3) tan θ < 0, sin θ > 0
4) cot θ and csc θ are positive
5) sec θ > 0, csc θ < 0
6) cot θ < 0, cos θ < tan θ

B. Determine the reference angle, θ_R, for the following:

1) 205°
2) 60°
3) −97°
4) 385°30′
5) 1000°
6) 102°
7) 275°
8) −500°
9) 91°
10) 5000°

C. Determine the exact value of the following:

1) sin 210°
2) sec 330°
3) cot 315°
9) sin 1035°
10) cos 495°
11) sin (−60°)

4) sin 150°

5) cos 225°

6) cos 450°

7) sec 390°

8) cot 690°

12) tan (−120°)

13) cot (−315°)

14) sec (−330°)

15) cot (−495°)

16) sin (−90°)

SOLUTIONS

A. (1) Quadrant IV (2) Quadrant III (3) Quadrant II (4) Quadrant I (5) Quadrant IV (6) Quadrant II

B. (1) 25° (2) 60° (3) 83° (4) 25°30′ (5) 80° (6) 78° (7) 85° (8) 40° (9) 89° (10) 40°

C. (1) −½ (2) $+\frac{2\sqrt{3}}{3}$ (3) −1 (4) +½ (5) $-\frac{\sqrt{2}}{2}$ (6) 0 (7) $+\frac{2\sqrt{3}}{3}$ (8) $-\sqrt{3}$ (9) $-\frac{\sqrt{2}}{2}$ (10) $-\frac{\sqrt{2}}{2}$ (11) $-\frac{\sqrt{3}}{2}$ (12) $+\sqrt{3}$ (13) +1 (14) $+\frac{2\sqrt{3}}{3}$ (15) +1 (16) −1