VANCOUVER COMMUNITY COLLEGE

Conic Sections:

The Parabola

In earlier courses, you've looked at the parabola as the set of solutions to a quadratic equation. Now, you will start to see it as one of the **conic sections**. You'll learn about the centre of the parabola, the **focus**, and a line that has special properties related to the focus, the **directrix**.

FORMULAS FOR PARABOLAS

When the vertex of the parabola is at the origin, the formula is simple. For a parabola with a vertical line of symmetry (one that opens up or down), x is squared. For a parabola with a horizontal line of symmetry (one that opens right or left), y is squared.

Standard form:	$x^2 = 4py$	$y^2 = 4px$
Vertex:	(0, 0)	(0, 0)
Focus:	(0, p)	(p, 0)
Directrix:	$y = -p$	$x = -p$

When the vertex is not at origin, then the parabola has been translated (it has undergone a shift or a slide).

Standard form:	$(x - h)^2 = 4p(y - k)$	$(y - k)^2 = 4p(x - h)$
Vertex:	(h, k)	(h, k)
Focus:	(h, k + p)	(h + p, k)
Directrix:	$y = k - p$	$x = h - p$

Example 1: Determine the vertex, focus and directrix of the parabola $x^2 + 8x + 8y = 8$.

Solution: First we need to get the parabola into the standard form. We can tell that the parabola is a vertical parabola, since there is an x^2 term instead of a y^2 term. To convert our equation to standard form, we start by completing the square.

$$\begin{aligned} x^2 + 8x + 8y &= 8 \\ x^2 + 8x &= 8 - 8y \\ x^2 + 8x + \left(\tfrac{8}{2}\right)^2 &= 8 - 8y + \left(\tfrac{8}{2}\right)^2 \\ x^2 + 8x + 16 &= 24 - 8y \\ (x + 4)^2 &= 24 - 8y \end{aligned}$$

Now we need to format the right-hand side so that it matches standard form. We factor out 4p from both terms:

$$\begin{aligned} (x + 4)^2 &= -8(y - 3) \\ &= 4 \cdot (-2)(y - 3) \\ \therefore p = -2, h &= -4, k = 3 \end{aligned}$$

The vertex is at (−4, 3), the focus is at (−4, 3 − 2) = (−4, 1) and the directrix is y = 3 − (−2) or y = 5.

Authored by Gordon Wong

EXERCISES

A. Determine the vertex, focus and directrix of the following parabolas:

1) $x^2 = 4y$
2) $x^2 = 4y + 8$
3) $y^2 = 8x + 2$
4) $(x - 3)^2 = 3y + 6$
5) $x + y^2 + 4y = 0$
6) $y^2 - 2y + 4x - 4 = 0$

B. Find the equation of the parabola that satisfies these conditions. Give your answers in general (i.e., simplified) form:

1) vertex is at (0, 0); focus at (0, 4)
2) vertex is at (1, 0); directrix at $x = 0$
3) focus at (1, 4); directrix at $y = 2$

SOLUTIONS

A. (1) vertex: (0, 0); focus: (0, 1); directrix: $y = -1$
(2) vertex: (0, −2); focus: (0, −1); directrix: $y = -3$
(3) vertex: $(-\frac{1}{4}, 0)$; focus: $(\frac{7}{4}, 0)$; directrix: $x = -\frac{9}{4}$
(4) vertex: (3, −2); focus: $(3, -\frac{5}{4})$; directrix: $y = -\frac{11}{4}$
(5) vertex: (4, −2); focus: $(\frac{15}{4}, -2)$; directrix: $x = \frac{17}{4}$
(6) vertex: $(\frac{5}{4}, 1)$; focus: $(\frac{1}{4}, 1)$; directrix: $x = \frac{9}{4}$

B. (1) $x^2 = 16y$ (2) $y^2 = 4x - 4$ (3) $(x - 1)^2 = 4y - 12 \rightarrow x^2 - 2x - 4y = -13$