



The Box Method

of Graphing Sinusoid Functions

By now, you've done transformations of parabolas and other functions. These transformations can be applied to sine waves and cosine waves, which we often see in physics. These transformations are so common that there's new vocabulary to learn about each aspect of the transformation.

Since cosine waves and sine waves are basically the same shape, we'll focus on sine waves, but all the concepts in this worksheet apply to both. A basic transformation of a sine wave looks like this:

$$y = \pm a \sin [b(x - h)] + k$$

CHANGE IN PERIOD:
how much the wave
travels before it repeats

vertical
shift

AMPLITUDE:
how far above and
below the "centre
line" the wave gets

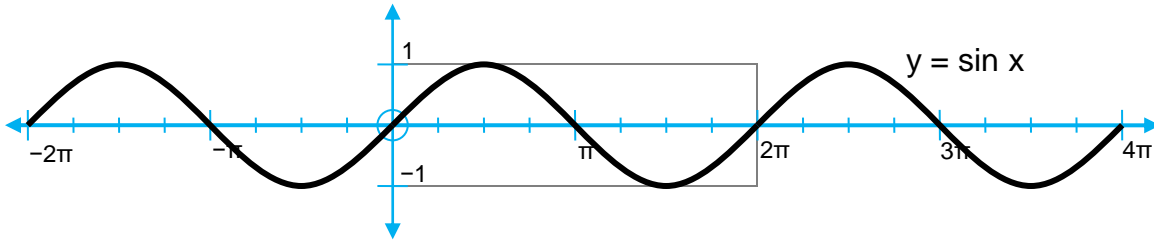
PHASE SHIFT:
horizontal shift

The variables a , b , h and k still do the same things: anything inside the [brackets] affects the graph horizontally (and has the opposite effect of what it looks like it should); anything outside the [brackets] affects the graph vertically. Additions and subtractions are shifts. Multiplications are stretches or squishes; multiplying by negative numbers flips the graph over. Because the graphs of sine functions are **periodic** (they repeat themselves endlessly), b won't often be negative.

Transformations of sine waves can be hard to do because we may need to draw several versions of the same curve on the same axes, and that's messy and confusing. The Box Method for graphing sine waves helps with this by allowing you to draw the final curve on the first try. Instead of drawing the curve itself, you draw a frame (or box) for the curve to fit in.

We'll start by looking at an un-transformed sine wave. The graph oscillates around the x -axis. It has an amplitude of 1 (i.e. it goes from -1 to $+1$ in the y direction) and a period of 2π (it starts to repeat itself after $x = 2\pi$). The sine wave starts at origin, heading up. Notice also that the curve reaches a maximum value one-quarter of the way through its period, crosses the x -axis again halfway through its period and reaches a minimum value three-quarters of the way through its period. We'll use all this information in making our graphs of transformations.





We can draw a bounding box around the “first” period of this graph, from 0 to 2π , including a horizontal line through the centre that indicates the axis that the wave oscillates around. For a wave that’s been transformed, if you can figure out where the bounding box ends up after the transformation, you can easily draw the curve inside it, and repeat it as often as necessary.

Example 1: Graph $y = 2 \sin (2x - \pi/2) + 1$.

Solution: First, we need the function equation in the form as it appears on page 1: $y = a \sin [b(x - h)] + k$. To do this, the terms inside the bracket need to be factored:

$$\begin{aligned} y &= 2 \sin (2x - \pi/2) + 1 \\ &= 2 \sin [2(x - \pi/4)] + 1 \end{aligned}$$

So $a = 2$, $b = 2$, $h = \pi/4$ and $k = 1$. We’ll use these numbers to draw the bounding box for the “first period”.

First, use the phase shift to find the left edge of the box. The phase shift is $\pi/4$. The original sine wave starts at 0, so ours will start at $\pi/4$. In light pencil, we draw a vertical line on our graph at $x = \pi/4$.

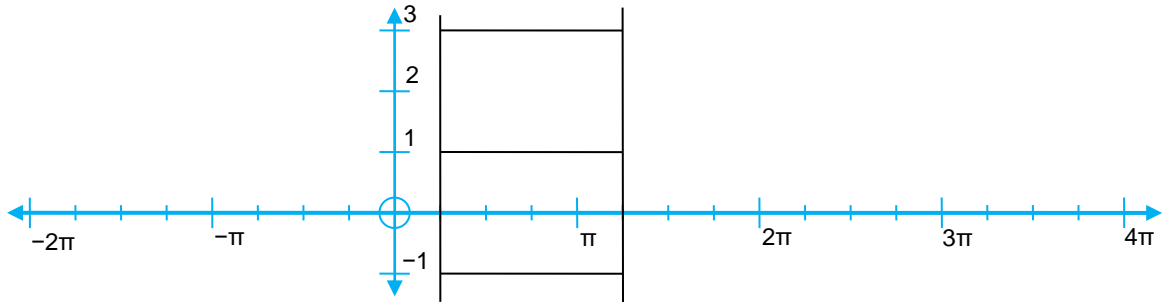
Second, use the period length to find the right edge of the box. The new period can be calculated by $2\pi \div b$. For our curve, this is $2\pi \div 2 = \pi$. The sine wave repeats every time x is increased by π . Since it starts at $\pi/4$, the first period ends at $\pi/4 + \pi = 5\pi/4$. We draw a vertical line on our graph at $x = 5\pi/4$. If the question requires us to draw multiple periods, we could keep adding π to the location of the first line to make more boxes.

The period may appear negative. For $y = 2 \sin (-2x - \pi/2) + 1$, “ b ” would be -2 , which would be strange. We can correct this using the identities $\sin (-x) = -\sin (x)$ and $\cos (-x) = \cos x$. For sine, this means we can use the inverted sine wave $y = -\sin x$ (which starts from origin and goes *down* first), and for cosine, we can ignore the effects of a minus sign on “ b ”.

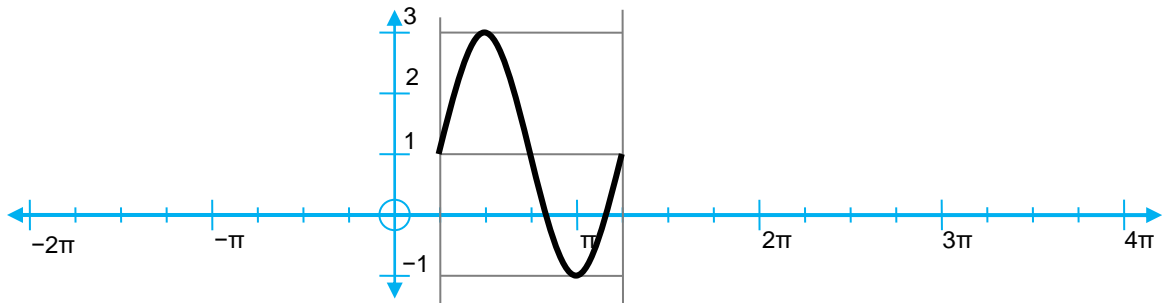
Third, use the vertical shift to draw a new centre line. Our sine wave has been shifted upwards by 1. The old centre line was at $y = 0$, so the new one is at $y = 1$. Draw a horizontal line at $y = 1$ for every period of the curve you intend to draw.

Fourth, use the amplitude to draw the top and bottom of the box. The amplitude of our curve is 2, so the new amplitude is $2 \times 1 = 2$ units. (If the amplitude appears negative, it has the effect of flipping the curve vertically. Amplitude is a distance, so it’s always positive.) Draw the top of the box 2 units above the centre line (at $y = 3$) and the bottom of the box 2 units below the centre line (at $y = -1$). Your graph should now look something like this:





Last, draw the curve in the box. The standard sine wave divides neatly into quarters. It starts at the centre for the left edge of the box, then goes up to the top, down to the centre, down to the bottom, and back up to the centre. Usually, the quarter marks for the box line up with marks on the axis, making the curve easier to draw. Now we can draw a standard sine wave into this box, and then erase the box if need be. We can also make more boxes to continue the pattern. Check with your instructor to see whether the first period of the curve is enough of an answer, or whether more is required for full marks.



EXERCISES

A. If the graph of $y = \sin x$ can be described as “centre, top, centre, bottom, centre”, then what five words describe the graph of:

1) $y = \cos x$?

2) $y = -\sin x$?

B. Determine the (a) phase shift, (b) period, (c) locations of the left and right edges of the bounding box, (d) amount of vertical shift, (e) location of the centre line, (f) amplitude, and (g) locations of the top and bottom of the bounding box.

1) $y = \sin \left[\frac{1}{3}(x + \frac{\pi}{2}) \right] - 1$

2) $y = 2 \sin \left[-\frac{1}{2}(x - \frac{\pi}{4}) \right]$

3) $y = \frac{1}{2} \cos (-2x + \pi) - \frac{5}{2}$

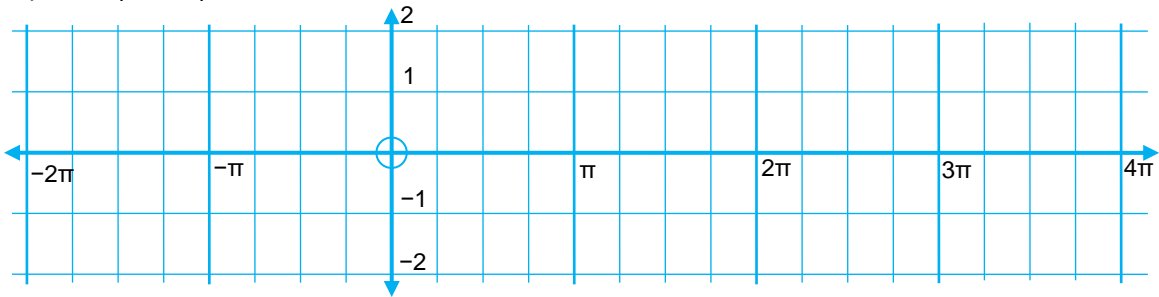
4) $y = -3 \sin \left(x - \frac{3\pi}{4} \right) - 2$

5) $y = 4 \sin (\pi x + \pi) + 6$

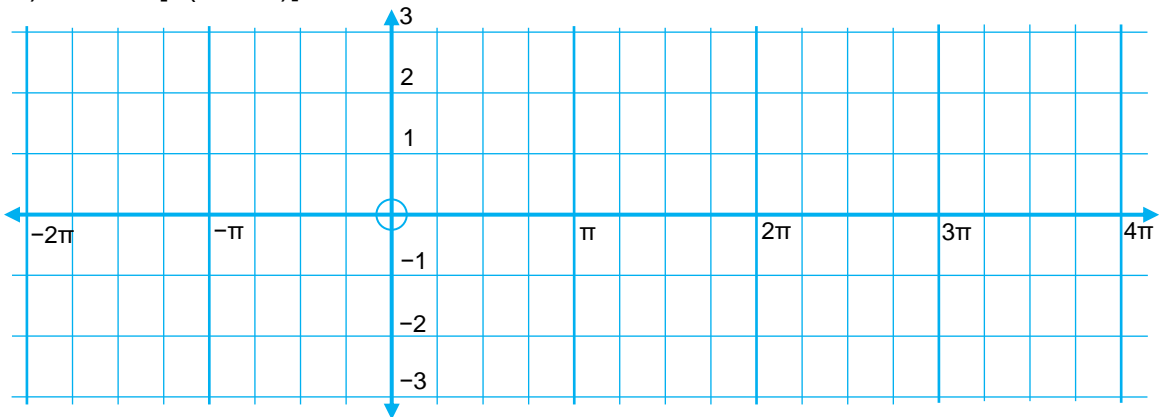


C. Graph the following.

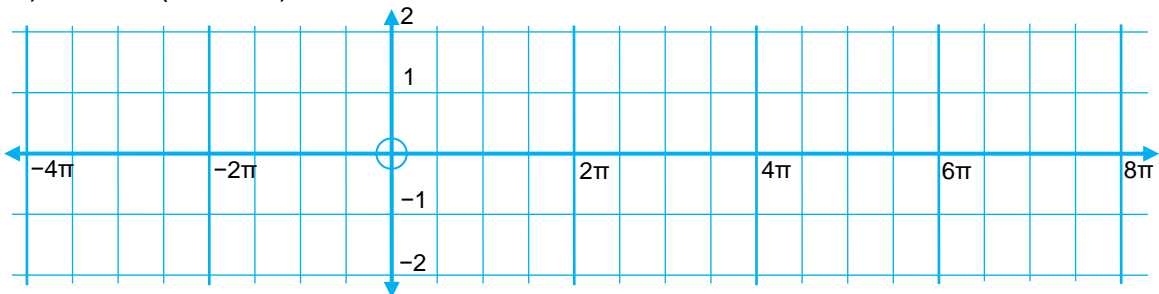
1) $\sin(x - \pi) - 1$



2) $-2 \sin[2(x + \pi/4)] + 1$



3) $\frac{1}{2} \cos(\frac{1}{3}x + \pi/6) + \frac{1}{2}$



SOLUTIONS

A: 1) top, centre, bottom, centre, top 2) centre, bottom, centre, top, centre

B: 1) (a) $-\pi/2$ (b) 6π (c) $x = -\pi/2, x = 11\pi/2$ (d) -1 (e) $y = -1$ (f) 1 (g) $y = 0, y = -2$

2) (a) $\pi/4$ (b) 4π (c) $x = \pi/4, x = 17\pi/4$ (d) 0 (e) $y = 0$ (f) 2 (g) $y = -2, y = 2$

3) (a) $-\pi/2$ (b) π (c) $x = -\pi/2, x = \pi/2$ (d) $-5/2$ (e) $y = -5/2$ (f) $1/2$ (g) $y = -3, y = -2$

4) (a) $3\pi/4$ (b) 2π (c) $x = 3\pi/4, x = 11\pi/4$ (d) -2 (e) $y = -2$ (f) 3 (g) $y = -5, y = 1$

5) (a) -1 (b) 2 (c) $x = -1, x = 1$ (d) 6 (e) $y = 6$ (f) 4 (g) $y = 2, y = 10$

C:

