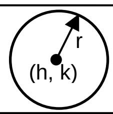
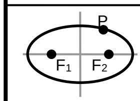
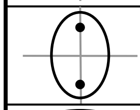
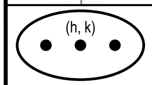
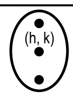
VANCOUVER COMMUNITY COLLEGE

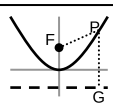
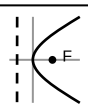
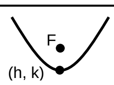
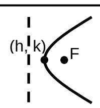
Conic Sections Review

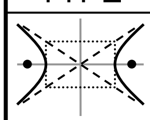
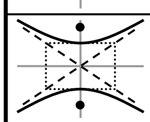
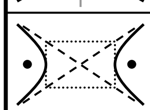
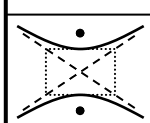
CIRCLE			
TYPE	GEN. FORMULA	CENTRE	RADIUS
	$(x-h)^2+(y-k)^2=r^2$	(h, k)	r

DISTANCE FORMULA $\overline{P_1P_2}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

MIDPOINT FORMULA $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

ELLIPSE					
TYPE	GEN. FORMULA	CENTRE	VERTICES	FOCI	RELATIONSHIPS
	$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$	$(0, 0)$	$(a, 0)$, $(-a, 0)$	$(c, 0)$, $(-c, 0)$	$\overline{PF_1}+\overline{PF_2}=2a$
	$\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1$	$(0, 0)$	$(0, a)$, $(0, -a)$	$(0, c)$, $(0, -c)$	$c^2=a^2-b^2$
	$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$	(h, k)	$(h+a, k)$, $(h-a, k)$	$(h+c, k)$, $(h-c, k)$	$2a$ = major axis = longer axis; $2b$ = minor axis = shorter axis
	$\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1$	(h, k)	$(h, k+a)$, $(h, k-a)$	$(h, k+c)$, $(h, k-c)$	$a>b>0$

PARABOLA							
TYPE	GEN. FORMULA	VERTEX	DIRECTRIX	FOCUS	LINE OF SYMMETRY	RELATIONSHIPS	
	$x^2=4py$	$(0, 0)$	$y=-p$	$(0, p)$	$x=0$	∪ if $p>0$; ∩ if $p<0$	$\overline{PF}=\overline{PG}$
	$y^2=4px$	$(0, 0)$	$x=-p$	$(p, 0)$	$y=0$	⊂ if $p>0$; ⊃ if $p<0$	
	$(x-h)^2=4p(y-k)$	(h, k)	$y=-p+k$	$(h, p+k)$	$x=h$	∪ if $p>0$; ∩ if $p<0$	
	$(y-k)^2=4p(x-h)$	(h, k)	$x=-p+h$	$(p+h, k)$	$y=k$	⊂ if $p>0$; ⊃ if $p<0$	

HYPERBOLA							
TYPE	GEN. FORMULA	CENTRE	ASYMPTOTES	FOCUS	LINE OF SYMMETRY	VERTICES	RELATIONSHIPS
	$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$	$(0, 0)$	$y=\pm\dfrac{b}{a}x$	$(c, 0)$, $(-c, 0)$	$y=0$	$(a, 0)$, $(-a, 0)$	$\overline{PF_1}-\overline{PF_2}=2a$; $c^2=a^2+b^2$
	$\dfrac{y^2}{a^2}-\dfrac{x^2}{b^2}=1$	$(0, 0)$	$y=\pm\dfrac{a}{b}x$	$(0, c)$, $(0, -c)$	$x=0$	$(0, a)$, $(0, -a)$	$\overline{PF_1}-\overline{PF_2}=2a$; $c^2=a^2+b^2$
	$\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$	(h, k)	$y-k=\pm\dfrac{b}{a}(x-h)$	$(h+c, k)$, $(h-c, k)$	$y=k$	$(h+a, k)$, $(h-a, k)$	$\overline{PF_1}-\overline{PF_2}=2a$; $c^2=a^2+b^2$
	$\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$	(h, k)	$y-k=\pm\dfrac{a}{b}(x-h)$	$(h, k+c)$, $(h, k-c)$	$x=h$	$(h, k+a)$, $(h, k-a)$	$\overline{PF_1}-\overline{PF_2}=2a$; $c^2=a^2+b^2$

EXERCISES

A. Identify the shape of each graph, and supply these details:
CIRCLE: centre and radius   PARABOLA: vertex, focus, directrix and line of symmetry
ELLIPSE: centre, vertices and foci   HYPERBOLA: centre, vertices, foci and asymptotes

1) $x^2 + y^2 = 15$

2) $x = (y + 1)^2 - 3$

3) $3x^2 = y^2 + 9$

4) $4x^2 + 9y^2 = 36$

5) $x^2 + y^2 - 8x + 4y + 4 = 0$

6) $y = 3(x - 4)^2 - 2$

7) $4x^2 - 3y^2 = -12$

8) $y^2 - 5x = 0$

9) $x^2 = 3y$

10) $25x^2 + 3y^2 = 75$

11) $9x^2 - 54x + 5y^2 + 10y + 41 = 0$

12) $4x^2 + 8x + 9y^2 + 36y + 4 = 0$

13) $3x^2 + 6x - 5y^2 + 50y = 137$

14) $y^2 + 2y - 2x^2 - 4x = 3$

B. Determine:

1) the equation of the circle with (−2, 3) and (4, −1) as endpoints of its diameter.

2) the radius and the equation of the circle having a centre (5, −2) and passing through the point (4, −3).

3) the focus of a parabola having a directrix x = 3 and a vertex (1, 4).

4) the equation of an ellipse with vertices (±4, 0) and containing the point $(3, \frac{\sqrt{14}}{4})$.

5) the equation of a hyperbola having vertices at (±3, 0) and foci at $(\pm\sqrt{13}, 0)$.

6) the equation of a parabola with a focus (0, −1) and a directrix, y = −3.

7) the equation of a hyperbola having asymptotes 5x − 4y = 0 and 5x + 4y = 0 and a vertex at $(\sqrt{41}, 0)$.

8) the equation of an ellipse with centre at (−4, 3), with a major axis of length 5, parallel to the y-axis, and a minor axis of length 3.

C. 1) Graph the hyperbola $\frac{y-1}{16}-\frac{x+3}{9}=1$. Include the rectangle used to define the asymptotes of the hyperbola, with its vertices at (h ± b, k ± a). (Call this value of b "b_h".)

2) Determine c, the distance from the centre of the hyperbola to either focus. (Call this value c_h.)

3) What is the equation of the circle with the same centre as the hyperbola in (1) and a radius of c_h? Graph this circle on the same axes as in (1).

4) What is the equation of the ellipse with the same centre as the hyperbola in (1) with a major axis equal to $2c_h$ and a minor axis equal to $2b_h$? Graph this ellipse on the same axes as in (1). Include the foci.

5) What points of interest are shared among the three curves?

SOLUTIONS

A. (1) circle: C (0,0) R: $\sqrt{15}$ (2) parabola: V (−3, −1) F ($\frac{-11}{4}$, −1) D: x = $\frac{-13}{4}$; LoS: y = −1 (3) hyperbola: C (0, 0) V ($\pm\sqrt{3}$, 0) F ($\pm\sqrt{12}$, 0) A: y = $\pm\sqrt{3}$x (4) ellipse: C (0, 0) V (±3, 0) F ($\pm\sqrt{5}$, 0) (5) circle: C (4, −2) R: 4 (6) parabola: V (4, −2) F (4, $-\frac{23}{12}$) D: y = $-\frac{25}{12}$; LoS: x = 4 (7) hyperbola: C (0, 0) V (0, ±2) F (0, $\pm\sqrt{7}$) A: y = $\pm\frac{2\sqrt{3}}{3}$x (8) parabola: V (0, 0) F ($\frac{5}{4}$, 0) D: x = $-\frac{5}{4}$ LoS: y = 0 (9) parabola: V (0, 0) F (0, $\frac{3}{4}$) D: y = $-\frac{3}{4}$; LoS: x = 0 (10) ellipse: C (0, 0) V (0, ±5) F (0, $\pm\sqrt{22}$) (11) ellipse: C (3, −1) V (3, −4), (3, 2) F (3, −3), (3, 1) (12) ellipse: C (−1, −2) V (2, −2), (−4, −2) F (−1 ± $\sqrt{5}$, −2) (13) hyperbola: C (−1, 5) V (−1 ± $\sqrt{5}$, 5) F (−1 ±2$\sqrt{2}$, 5) A: y − 5 = ±$\frac{\sqrt{15}}{5}$(x + 1) (14) hyperbola: C (−1, −1) V (−1, −1 ±$\sqrt{2}$) F (−1, −1 ±$\sqrt{3}$) A: y + 1 = ±$\sqrt{2}$(x + 1)

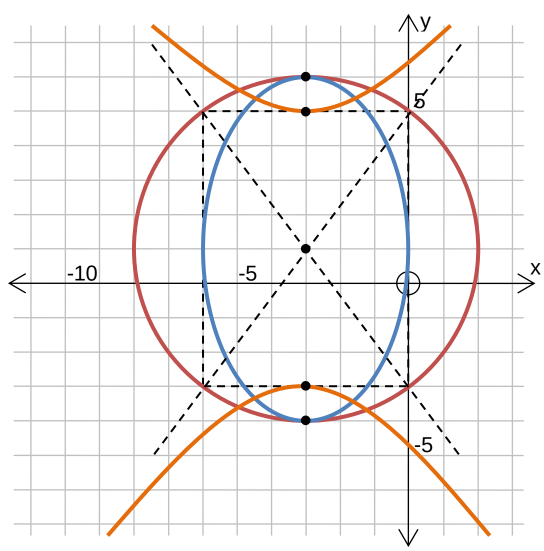
B. (1) $(x-1)^2+(y-1)^2=13$ (2) R: $\sqrt{2}$; $(x-5)^2+(y+2)^2=2$ (3) (−1, 4) (4) $\frac{x^2}{16}+\frac{y^2}{2}=1$ (5) $\frac{x^2}{9}-\frac{y^2}{4}=1$ (6) $x^2=4(y+2)$ (7) $\frac{x^2}{41}-\frac{16y^2}{1025}=1$ (8) $\frac{4(x+4)^2}{9}+\frac{4(y-3)^2}{25}=1$

C. (1) See graph. (2) $c_h = 5$

(3) $(x+3)^2+(y-1)^2=25$. See graph.

(4) $\frac{(x+3)^2}{9}+\frac{(y-1)^2}{25}=1$. See graph.

(5) All three curves have the same centre by design. The circle passes through the foci of the hyperbola and all four corners of the rectangle. The vertices of the ellipse are the same points as the foci of the hyperbola, and the vertices of the hyperbola are the same points as the foci of the ellipse. The vertices of the ellipse also lie on the circle. (This is no coincidence. These relationships between the circle, ellipse and hyperbola will be true no matter what hyperbola you start with.



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