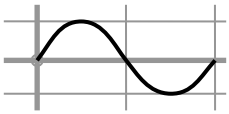
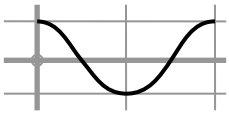
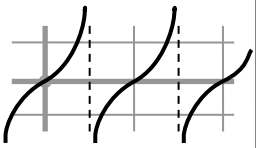
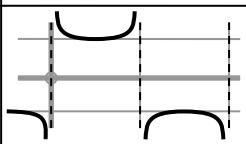
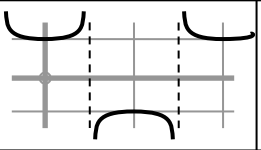
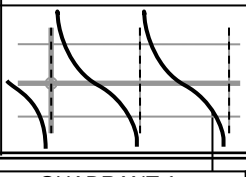
The Circular Functions

FUNCTION	GRAPH	DOMAIN	RANGE	PERIOD
sine (odd fn)		all real numbers	$[-1, 1]$	2π
cosine (even fn)		all real numbers	$[-1, 1]$	2π
tangent (odd fn)		all real numbers except $\pi/2 + k\pi$	all real numbers	π
cosecant (odd fn)		all real numbers except $k\pi$	$(-\infty, -1] \cup [1, \infty)$	2π
secant (even fn)		all real numbers except $\pi/2 + k\pi$	$(-\infty, -1] \cup [1, \infty)$	2π
cotangent (odd fn)		all real numbers except $k\pi$	all real numbers	π

FORMULAS

$\sin x = \sin(x + 2k\pi)$ $\quad$ $\cos x = \cos(x + 2k\pi)$

$\sin(-x) = -\sin x$ $\quad$ $\cos(-x) = \cos x$

$\sin(x \pm \pi) = -\sin x$ $\quad$ $\cos(x \pm \pi) = -\cos x$

$\sin(\pi - x) = \sin x$ $\quad$ $\cos(\pi - x) = -\cos x$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x} \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

PYTHAGOREAN

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$1 + \tan^2 x = \sec^2 x$

COFUNCTION IDENTITIES

$\sin(x \pm \pi/2) = \pm \cos x$

$\cos(x \pm \pi/2) = \mp \sin$

degrees $\xrightarrow{\times \pi/180}$ radians $\xrightarrow{\times 180/\pi}$ degrees

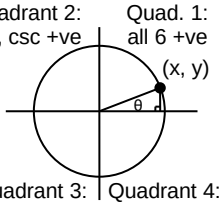
EVEN & ODD FUNCTIONS

even: $f(x) = f(-x)$

odd: $f(-x) = -f(x)$

Quadrant 2: sin, csc +ve

Quad. 1: all 6 +ve



Quadrant 3: tan, cot +ve

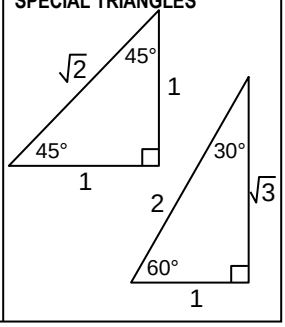
Quadrant 4: cos, sec +ve

$\sin\theta = y/r$ $\quad \csc\theta = r/y$

$\cos\theta = x/r$ $\quad \sec\theta = r/x$

$\tan\theta = y/x$ $\quad \cot\theta = x/y$

SPECIAL TRIANGLES



	QUADRANT 1					QUADRANT 2				QUADRANT 3				QUADRANT 4				
degrees θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$\sin\theta$																	
$\cos\theta$																	
$\tan\theta$																	
$\csc\theta$																	
$\sec\theta$																	
$\cot\theta$																	

*** On this page, k represents any integer.

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EXERCISES

A. Complete the table at the bottom of the other side of the page.

B. Label the axes in the table at the top left of the first page. What values do the thin grey lines represent?

C. A value of $\frac{\pi}{5}$ rad determines a point on the unit circle with coordinates $\left(\frac{1+\sqrt{5}}{4}, \frac{\sqrt{10-2\sqrt{5}}}{4}\right)$, or about (.809, .587). From this information and the formulas on the other side of this page, determine:

1) sin $\pi/5$
2) cos $\pi/5$
3) tan $\pi/5$
4) coordinates for $4\pi/5$
5) sec $6\pi/5$
6) cos $101\pi/5$
7) sin $3\pi/10$ [*Hint:* $4/5 - 1/2 = 3/10$]
8) cot $7\pi/10$

D. In which quadrant is:

1) sin and tan positive?
2) sin and tan negative?
3) cos positive and tan negative?
4) cot positive and sec negative?

E. What is the greatest value sec x can have in Quadrant 4?

F. Does 7 csc x = 2 have a solution for x? Explain.

G. Expand:

1) $(\sin x - \cos x)(\sin x + \cos x)$
2) $(\sin x + \cos x)^2$

H. Factor, and use trigonometric identities to simplify, if possible:

1) $\sin^4 x - \cos^4 x$
2) $\sin^3 x \cos^2 x + \sin^2 x \cos^3 x + \sin x \cos^2 x$
3) $\cot^2 \theta - 4 \cot \theta - 12$

I. Verify the following identities:

1) $\csc x - \sin x = \cos x \cot x$
2) $\cot x \sec^2 x - \cot x = \tan x$
3) $\tan^2 x \cos^2 x + \cot^2 x \sin^2 x = 1$

SOLUTIONS

B. horizontal lines: y = −1 and y = 1; vertical lines: x = π and x = 2π

C. (1) .587 (2) .809 (3) .726 (4) (−.809, .587) (5) −1.236 (6) .809 (7) .809 (8) $\frac{\cos(\pi/5+\pi/2)}{\sin(\pi/5+\pi/2)} = \frac{-\sin(\pi/5)}{\cos(\pi/5)}$ = −.726 D. (1) Quad. 1 (2) Quad. 4 (3) Quad. 4 (4) Quad. 3

E. ∞ F. csc x = 2/7 does not have a solution, since csc cannot have values between −1 and 1. G. (1) $\sin^2 x - \cos^2 x$ (2) $1 + 2 \sin x \cos x$ H. (1) $(\sin x + \cos x)(\sin x - \cos x)$ (2) $\sin x \cos^2 x(\sin^2 x + \sin x \cos x + 1)$ (3) $(\cot \theta - 6)(\cot \theta + 2)$

I. Convert everything to sin and cos. Look for common pieces of trigonometric identities for clues as to how to proceed. Many solutions are possible.