



Conic Sections: The Circle

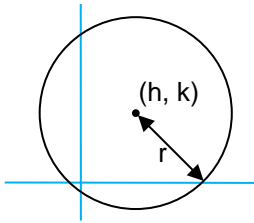
The equation for a circle has an x^2 and y^2 term with the same sign, and the coefficients in front of those terms are equal.

FORMULA FOR A CIRCLE

Standard form:

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{centre: } (h, k)$$

radius: r



When the equation is written in standard form, it is easy to find the coordinates of the centre and the length of the radius. You can also create the equation of the circle when given the centre coordinates and radius.

Example 1: Find the equation of the circle with a centre at $(2, -5)$ and a radius of 7.

Solution: If the centre is at $(2, -5)$, then $h = 2$ and $k = -5$. The radius, r , is 7. We can plug these numbers directly into the equation:

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ (x - 2)^2 + (y - (-5))^2 &= 7^2 \\ (x - 2)^2 + (y + 5)^2 &= 49 \end{aligned}$$

Example 2: Find the centre and radius for the circle with the equation $x^2 + 6x + y^2 + 8y = 11$.

Solution: First we need to get the equation into the standard form. We start by completing the squares for x and for y (and adding the balancing terms to the right hand side).

$$\begin{aligned} (x^2 + 6x) + (y^2 + 8y) &= 11 \\ (x^2 + 6x + 9) + (y^2 + 8y + 16) &= 11 + 9 + 16 \\ (x + 3)^2 + (y + 4)^2 &= 36 \\ (x + 3)^2 + (y + 4)^2 &= 6^2 \end{aligned}$$

Now we can see h , k and r : $h = -3$, $k = -4$ and $r = 6$. This means that the centre of the circle is at $(-3, -4)$ and the radius is 6.



EXERCISES

A. Find the centre and radius of each circle:

1) $(x + 1)^2 + (y + 5)^2 = \frac{1}{4}$

5) $x^2 - 10x + y^2 - 8y = 8$

2) $(x - 3)^2 + (y + 3)^2 = 3$

6) $2x^2 + 4x + 2y^2 + 8y = 2$

3) $x^2 = 64 - y^2$

7) $3x^2 - 24x + 3y^2 + 6y = 15$

4) $x^2 + 4x + y^2 + 6y = 3$

B. Find an equation of a circle in standard form with the given centre and radius:

1) centre: (1, 1), radius: 3

3) centre: (-1, 0), radius: $2\sqrt{5}$

2) centre: (3, -5), radius: $\sqrt{3}$

4) centre: $(\frac{1}{2}, \frac{1}{2})$, radius: $\frac{1}{4}$

C. Draw the circle described by the following equation:

$(x - 2)^2 + (y - 2)^2 = 16$

SOLUTIONS

A: (1) centre: (-1, -5), radius: $\frac{1}{2}$ (2) centre: (3, -3), radius: $\sqrt{3}$

(3) centre: (0, 0), radius: 8 (4) centre: (-2, -3), radius: 4

(5) centre: (5, 4), radius: 7 (6) centre: (-1, -2), radius: $\sqrt{6}$

(7) centre: (4, -1), radius: $\sqrt{22}$

B: (1) $(x - 1)^2 + (y - 1)^2 = 9$ (2) $(x - 3)^2 + (y + 5)^2 = 3$

(3) $(x + 1)^2 + y^2 = 20$ (4) $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{16}$

C: See diagram.

