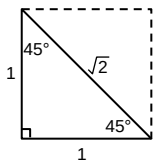
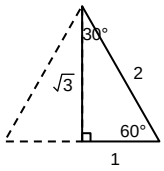
Advanced Trigonometry

SPECIAL TRIANGLES





SINE AND COSINE IDENTITIES

$\sin\theta = \sin(\theta + 2k\pi)$, k is an integer
$\sin(-\theta) = -\sin\theta$ *
$\sin(\theta \pm \pi) = -\sin\theta$
$\sin(\pi - \theta) = \sin\theta$

* This is true for odd functions like: cot, csc, tan

$\cos\theta = \cos(\theta + 2k\pi)$, k is an integer
$\cos(-\theta) = \cos\theta$ **
$\cos(\theta \pm \pi) = -\cos\theta$
$\cos(\pi - \theta) = -\cos\theta$

** This is true for other even functions like: sec

RECIPROCAL AND QUOTIENT IDENTITIES

$\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{1}{\tan\theta}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

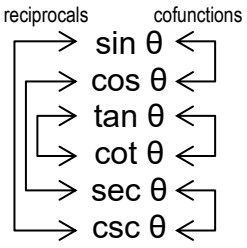
PYTHAGOREAN IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$
$1 + \cot^2\theta = \csc^2\theta$
$1 + \tan^2\theta = \sec^2\theta$

COFUNCTION IDENTITIES

$\sin\left(\theta \pm \frac{\pi}{2}\right) = \pm\cos\theta$
$\cos\left(\theta \pm \frac{\pi}{2}\right) = \mp\sin\theta$

RECIPROCALS AND COFUNCTIONS



Any trig function of a positive acute angle is equal to the *cofunction* of the complementary angle.
e.g. $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$
or $\sin\theta = \cos(90° - \theta)$

SUM & DIFFERENCE IDENTITIES

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

DOUBLE AND HALF ANGLE IDENTITIES

$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$
$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

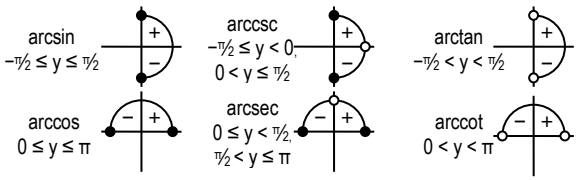
The choice of + or − in these three identities depends on which quadrant $\frac{\theta}{2}$ lies in

$\sin\frac{\theta}{2} = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}$
$\cos\frac{\theta}{2} = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}$
$\tan\frac{\theta}{2} = \pm\sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}}$
$= \dfrac{\sin\theta}{1 + \cos\theta}$
$= \dfrac{1 - \cos\theta}{\sin\theta}$

$\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$
$\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$
$\tan^2\theta = \dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}$

Authored by Gordon Wong

PRINCIPAL VALUES OF INVERSE FNS



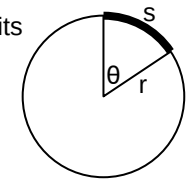
ANGULAR MOTION

$\theta = \frac{s}{r}$

$\omega = \frac{\theta}{t}$

$v = r\omega$

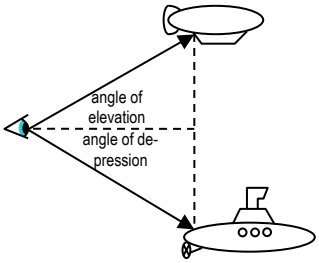
s = arc length } same units
r = radius }
θ = central angle (rad)
ω = angular speed
v = linear speed (at a point on rim)
t = time



DEGREES, MINUTES AND SECONDS

1° = 60′ 1′ = 60″

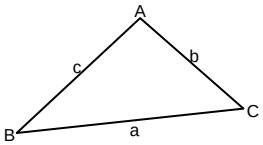
ANGLES OF ELEVATION AND DEPRESSION



SINE AND COSINE LAWS

Cosine Law (use with SAS or SSS)

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$



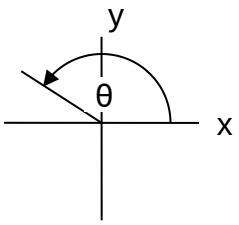
Sine Law (use with AAS, SSA)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

[Note: the SSA case may yield 0, 1 or 2 solutions.]

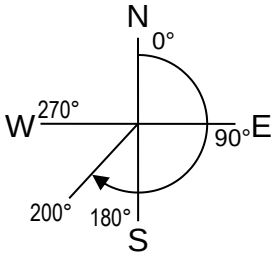
ANGLE SPECIFICATION TECHNIQUES

Directed Angles



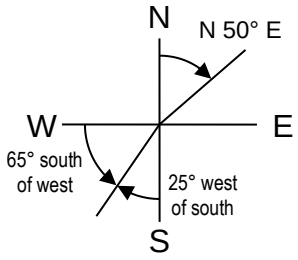
- used for trig class
- +ve angles: start at the positive x-axis and go CCW
- −ve angles: start at the positive x-axis and go CW
- angles higher than 360° or 2π rad are possible

Heading, Bearing or Course



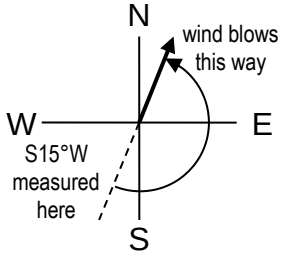
- used by airplanes and boats
- start at North, go CW
- angle of arrival (coming into a destination) is 180° away from the angle leaving a location (e.g., if angle of departure is 30°, angle of arrival will be 210°)

Compass Directions



- two forms:
  (1) "N 50° E" starts at North or South and moves CW or CCW eastward or westward
  (2) "65° south of west" starts at <u>*second*</u> compass point and moves CW or CCW toward the <u>*first*</u> compass point
- in form (2), every direction can be expressed two different ways

Wind Direction



- used in flying, weather reports
- wind direction is the direction the wind is blowing <u>*from*</u>, not <u>*to*</u>, e.g., a west wind blows from west to east
- find wind direction using above techniques; draw the vector pointing 180° away from the indicated angle