Tree Diagrams

When dealing with probability, it is helpful to think about the “sample space”, which is the complete list or representation of all the possible outcomes of an event. There are many ways that a “sample space” can be represented such as contingency tables, lists, pictures, or tree diagrams. This worksheet focuses on constructing, understanding and using tree diagrams. A tree diagram doesn’t actually look like a tree, but it uses the concept of “branches” and is a visual way to break down a calculation of probability for certain outcomes into several stages (or choices), where each branch in the tree represents one possible outcome (or one choice).

Let’s say you had 12 beers left over from a party. Nine of the beers are lagers, and three are ales. If you reach into the fridge without looking and grab a beer at random, what is the probability of getting a lager? What is the probability of getting a lager the 2nd time if you got a lager the 1st time? These are the types of questions that a tree diagram will help you to answer.

Let’s start constructing our tree diagram for this situation. We start on the left and move to the right as we build the diagram. The first part of the tree is a circle (or node) that represents the start of an event (the closed fridge = black circle). When you open the door and reach in, you have two possible outcomes: getting a lager or an ale. These outcomes are represented by branches. Since we have two outcomes, we draw two branches extending from the first node.

We’ll use “L” to represent lager and “A” to represent ale. At the tip of each branch, we write the probability of that particular outcome. The probability of getting a lager is given by:

\[
P(L) = \frac{\text{total number of lagers}}{\text{total number of beers}} = \frac{9}{12} = 0.75
\]

This is the probability we show above the “L” circle in the first branch. We do likewise for

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lager Lager</td>
<td>P(L and L) = (0.75)(0.73) ≈ 0.55</td>
</tr>
<tr>
<td>Lager IPA</td>
<td>P(L and A) = (0.75)(0.27) ≈ 0.20</td>
</tr>
<tr>
<td>IPA Lager</td>
<td>P(A and L) = (0.25)(0.82) ≈ 0.21</td>
</tr>
<tr>
<td>IPA IPA</td>
<td>P(A and A) = (0.25)(0.18) ≈ 0.05</td>
</tr>
</tbody>
</table>

We'll use “L” to represent lager and “A” to represent ale. At the tip of each branch, we write the probability of that particular outcome. The probability of getting a lager is given by:

\[
P(L) = \frac{9}{12} = 0.75
\]
the ale outcome and find P(A) = 3/12 = 0.25 for the first draw. It is helpful to label the column above the branch to indicate the event that is occurring (first draw, second draw, etc.).

The sum of probabilities for choosing a lager or ale in your first draw must add up to 1. As a rule for tree diagrams, the sum of probabilities of the branches extending from a single node must add up to 1.

On your second draw, there are again two possible outcomes, lager or ale, regardless of which beer was drawn the first time. So we draw two new branches extending from each of the nodes in the first stage. We could continue branching out from each circle in the diagram to represent picking a third beer, a fourth beer, and so on, determining the probability at each possible outcome.

To calculate the probability of getting a lager or ale on the second draw, recall that you’ve already pulled 1 beer out of the fridge (whether it was a lager or ale), so this reduces the total number of beers to 11. If the first beer you chose was a lager, you’ve also reduced the number of lagers available to grab by 1, so there are 8 lagers (and 4 ales) left when branching from the first L node. The probability of getting a lager the second time you reach into a fridge is 0.73 (8/11), given that you’ve already drawn a lager the first time. Note that after the first set of outcomes in the tree diagram, the probabilities are all conditional probabilities; in other words, the probability of the second outcome is affected by what happened in the first outcome.

Example 1: What is the probability of getting a lager first AND getting a lager second?
Solution: To determine this answer, we take the probabilities along the branches of the tree that describe those outcomes and multiply them together:

\[ P(L \text{ and } L) = 0.75 \times 0.73 = 0.55 \]

Example 2: What is the probability of getting an ale and a lager?
Solution: This question is a bit trickier because the order of drawing does not matter. So we could get a lager 1\text{st} and ale 2\text{nd} OR an ale 1\text{st} and lager 2\text{nd}. Since this is an “or” statement, we take the probabilities at the tips of the branches from the lager then ale path and the ale then lager path and add those probabilities together:

\[ P(L \text{ and } A) \text{ and } P(A \text{ and } L) = 0.2045 + 0.2045 \approx 0.41 \]

A tree diagram is not limited to branching twice from a node. For instance, if you had a bag of marbles with green, red and blue marbles inside and were asked to determine the probability of drawing a certain color, each node would branch three times as there are three possible colors you could get when drawing from the bag. It is also important to carefully read these types of problems to note whether an activity takes place with or without replacement as this will affect the total number of possible outcomes. This in turn affects the probability associated with a specific outcome. If in the example described above, you selected a beer with replacement, then the total number of beers available at each selection would always be 12.
Exercises

1. Twelve women and 14 men have applied for a job posting. You are responsible for interviewing applicants. If you select two applicants at random for an interview, what is the probability of that both of them are women (to 4 decimal places)?

2. A bag contains four green poker chips and ten blue poker chips. What is the probability when three chips are drawn without replacement that (a) all three chips will be green, (b) the first chip will be blue and the second and third chips will be green, (c) that two chips will be green and one chip will be blue? Round answers to three decimal places.

3. Micah has just graduated with a hospitality degree from VCC. She has applied for a job in Ontario at a four star hotel. The probability that she will get the job is 0.35. The probability that she will move to Ontario if she receives a job offer is 0.85. The probability that she will move to Ontario if she does not receive a job offer is 0.40. Create a tree diagram for all possible outcomes. What is the probability that she will be offered the job and not move to Ontario (to 4 decimal places)?

4. The owner of a local yoga studio tells you that the probability of a visitor buying a membership is 65%. The probability that someone will buy a membership and sign up for a yoga class is 26%. The probability that someone will not sign up for a class given that they did not buy a membership is 75%. (a) What is the probability that a visitor to the studio will sign up for a class, given that they bought a membership? (b) What is the probability that someone does not purchase a membership and signs up for a class?

5. There is a box of 12 donuts randomly arranged. Three donuts are chocolate glazed, five are maple glazed, and four are jelly-filled. You draw one donut from the box (without looking) and eat it. Then you draw a second donut from the box (without looking) and eat it. Construct a tree diagram for all the possible outcomes in the “experiment”. Label the probabilities of each stage of the experiment. (a) What is the probability of getting two chocolate glazed donuts? (b) What is the probability of getting a jelly-donut and a maple-glazed donut? (c) What is the probability of picking a chocolate donut given that you picked a maple donut on the first draw?
Solutions

1. \( P(\text{both applicants are women}) = \frac{12}{26} \cdot \frac{11}{25} = 0.2031 \)

2. (a) \( P(GGG) = 0.011 \)
   (b) \( P(BGG) = 0.055 \)
   (c) \( P(BGG) + P(GBG) + P(GGB) = 0.165 \)

3. \( P(\text{Job offer and not moving}) = 0.35 \cdot 0.15 = 0.0525 \)

4. (a) \( 40.0\% \) (The 26% is the joint probability from multiplying along the branches for this outcome. The conditional probability can be figured out by working backwards.)
   (b) \( 8.75\% \)

5. a) probability = 0.05
   b) probability = 0.15 + 0.15 = 0.30
   c) probability \( (C|M) = \frac{3}{11} = 0.27 \)