Ratio & Proportion

A proportion is an equality of two or more ratios:

\[
\frac{a}{b} = \frac{c}{d} \quad [1]
\]

In any proportion, the cross-products are equal:

\[ad = bc\]

Equation [1] could also be expressed using colon notation, as a ratio:

\[a : b : : c : d \quad \text{or} \quad a : b = c : d\]

In this example, a and d are called the **extremes** and b and c are called the **means**. Using this terminology:

the product of the extremes = the product of the means

\[ad = bc\]

**Example 1:** Solve: \(\frac{5}{25} = \frac{2}{x}\)

**Solution:** Set up the cross products:

\[5 \cdot x = 25 \cdot 2\]

Simplify:

\[5x = 50\]

Solve for x by dividing both sides by the coefficient of x:

\[5x \div 5 = 50 \div 5\]

\[x = 10\]

**Example 2:** Solve: \(\frac{2}{3} = \frac{\frac{1}{2}}{x}\)

**Solution:** Set up the cross products:

\[2 \cdot x = 3 \cdot \frac{1}{2}\]

Simplify:

\[2x = \frac{3}{2}\]

Solve for x. We could do this by dividing both sides by the coefficient of x, but since we have a fraction it might be easier to multiply both sides by the reciprocal of the coefficient (which amounts to the same thing). The reciprocal of 2 is \(\frac{1}{2}\), so:

\[2x \times \frac{1}{2} = \frac{3}{2} \times \frac{1}{2}\]

\[x = \frac{3}{4}\]
Example 3: If it costs 31 cents to cut 4 keys, how much would it cost to cut 12 keys?

Solution: Form a proportion for the problem. The units should match on each side of the equal sign (numerators should have the same units and so should the denominators):

\[
\frac{31 \text{ cents}}{4 \text{ keys}} = \frac{x \text{ cents}}{12 \text{ keys}}
\]

Set up the cross-products:

\[31 \cdot 12 = 4 \cdot x\]

Multiplying 31 \times 12 is hard. It is easier to divide by the coefficient now to make the calculations simpler:

\[(31 \cdot 12) \div 4 = 4x \div 4\]
\[31 \cdot 3 = x\]
\[93 = x\]

Write your answer to the question:

It would cost 93¢ to cut 12 keys.

Example 4: In Mark’s new job, he is paid $1700 every two weeks. What is Mark’s annual salary?

Solution: Form a proportion for the problem. The units should match on each side of the equal sign, so we have to convert between years and weeks to solve the problem:

\[
\frac{1700}{2 \text{ weeks}} = \frac{x}{1 \text{ year}}
\]
\[
\frac{1700}{2 \text{ weeks}} = \frac{x}{52 \text{ weeks}}
\]

Set up the cross-products:

\[1700 \cdot 52 = 2 \cdot x\]

Once again, it will be easier to divide by the coefficient now rather than later:

\[(1700 \cdot 52) \div 2 = 2x \div 2\]
\[1700 \cdot 26 = x\]
\[44,200 = x\]

Write your answer to the question:

Mark’s annual salary is $44,200.
EXERCISES

A. Solve for \( x \).

1) \( \frac{7}{24} = \frac{x}{48} \)

2) \( \frac{3}{4} = \frac{x}{60} \)

3) \( \frac{30}{5} = \frac{12}{x} \)

4) \( \frac{10}{5} = \frac{x}{7} \)

5) \( \frac{15}{x} = \frac{3}{6} \)

6) \( \frac{15}{x} = \frac{12}{4} \)

7) \( \frac{15}{x} = \frac{20}{4} \)

8) \( \frac{5}{20} = \frac{x}{32} \)

9) \( \frac{4}{8} = \frac{8}{x} \)

10) \( \frac{7}{52} = \frac{7}{x} \)

11) \( \frac{50}{75} = \frac{x}{\frac{1}{2}} \)

12) \( \frac{24}{72} = \frac{x}{9} \)

13) \( \frac{8}{x} = \frac{3}{9} \)

14) \( 7 : 3.5 = 21 : x \)

15) \( 15 : 25 = x : 100 \)

16) \( x : 4 :: 255 : 60 \)

17) \( 0.6 : x = 0.78 : 0.325 \)

18) \( 0.5 : 0.75 = x : 1 \)

19) \( 2 : 3 = \frac{1}{2} : x \)

20) \( \frac{7}{8} : \frac{3}{16} = x : \frac{5}{8} \)

21) \( 20 : 2.5 = x : \frac{1}{2} \)

22) \( 0.1 : 0.001 = 0.01 : x \)

B. Use a proportion to solve the following problems:

1) A supply of 500 halibut liver capsules sells for $4.80. What would be the price for 125 capsules?

2) In the Tofu Tasting Club, there are 45 women. The ratio of men to women is 3 to 5. How many men are there in the club?
3) If you had the uncanny ability of being able to crack open coconuts with your head at the rate of 4 coconuts every 9 minutes, how many coconuts could you crack open in 45 minutes?

4) A speedboat passes a race checkpoint 52.5 miles past the start of the course 2 hours after the race started. If the entire course is 210 miles long, how much time would you expect the speedboat to take to finish?

5) A manufacturing plant can make 750 microwave ovens in 9 days. How large an order for microwave ovens can the plant fill in 33 days?

6) A woman bought 180 acres of bog in Richmond for $51,000 and promptly sold 30 acres of it to you at cost. How much did she receive for your plot of land?

7) If you were to be paid $8.50 for two hours of work, how much should you receive in total for working a shift that is three hours longer?

8) If sound travels 825 metres in 2.5 seconds, how long will it take for the explosive sound of a cannon firing to travel 3.3 km? (Remember: 1 km = 1000 m)

9) 500 mL of liquid hand soap sells for $1.58. How much would 1.5 L of liquid hand soap cost? (Remember: 1 L = 1000 mL)

10) A special camera can take 500 pictures in 33 1/3 seconds. How many pictures can the camera take in 7 minutes of constant use?

SOLUTIONS

A. (1) 14  (2) 45  (3) 2  (4) 14  (5) 30  (6) 5  (7) 3  (8) 8  (9) 16  (10) 52  (11) 1/3 (12) 3  (13) 24  (14) 10.5  (15) 60  (16) 17  (17) .25  (18) .6  (19) 3/4  (20) 35/12  (21) 4  (22) 0.000 1

B. (1) $1.20  (2) 27 men  (3) 20 coconuts and 1 splitting headache  (4) 8 hours  (5) 2750 microwave ovens  (6) $8500  (7) $21.25 for 5 hours  (8) 10 seconds  (9) $4.74  (10) 6300 pictures