Advanced Trigonometry

**SINE AND COSINE IDENTITIES**

\[ \sin \theta = \sin (\theta + 2k\pi), \ k \text{ is an integer} \]
\[ \sin (-\theta) = -\sin \theta \]
\[ \sin (\theta \pm \pi) = -\sin \theta \]
\[ \sin (\pi - \theta) = \sin \theta \]

* These are the same for other odd functions.

\[ \cos \theta = \cos (\theta + 2k\pi), \ k \text{ is an integer} \]
\[ \cos (-\theta) = \cos \theta \]
\[ \cos (\theta \pm \pi) = -\cos \theta \]
\[ \cos (\pi - \theta) = -\cos \theta \]

** These are the same for other even functions.

**RECIPROCAL AND QUOTIENT IDENTITIES**

\[ \sec \theta = \frac{1}{\cos \theta}, \ \csc \theta = \frac{1}{\sin \theta}, \ \cot \theta = \frac{1}{\tan \theta} \]
\[ \tan \theta = \frac{\sin \theta}{\cos \theta}, \ \cot \theta = \frac{\cos \theta}{\sin \theta} \]

**PYTHAGOREAN IDENTITIES**

\[ \sin^2 \theta + \cos^2 \theta = 1 \]
\[ 1 + \cot^2 \theta = \csc^2 \theta \]
\[ 1 + \tan^2 \theta = \sec^2 \theta \]

**COFUNCTION IDENTITIES**

\[ \sin (\theta \pm \frac{\pi}{2}) = \pm \cos \theta \]
\[ \cos (\theta \pm \frac{\pi}{2}) = \mp \sin \theta \]

**RECIPROCALS AND COFUNCTIONS**

<table>
<thead>
<tr>
<th>Reciprocals</th>
<th>Cofunctions</th>
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<tbody>
<tr>
<td>( \sin \theta )</td>
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**PRINCIPAL VALUES OF INVERSE FNS**

\[ \arcsin \frac{-\pi}{2} \leq y \leq \frac{\pi}{2} \]
\[ \arccos 0 \leq y < \frac{\pi}{2} \]
\[ \arctan \frac{-\pi}{2} < y \leq \frac{\pi}{2} \]
\[ \arccot 0 < y < \frac{\pi}{2} \]

**SPECIAL TRIANGLES**

**SUM & DIFFERENCE IDENTITIES**

\[ \sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \]
\[ \cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \]
\[ \tan (\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \]

**DOUBLE AND HALF ANGLE IDENTITIES**

\[ \sin 2\theta = 2 \sin \theta \cos \theta \]
\[ \cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta \]
\[ \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \]

**IDENTITY FOR \( c \sin k\theta + b \cos k\theta \)**

\[ c \sin k\theta + b \cos k\theta = a \sin (k\theta + \beta) \]

where:

\[ a = \sqrt{b^2 + c^2} \]
\[ \beta = \tan^{-1} \frac{b}{c} \]

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ANGULAR MOTION
\[ \theta = \frac{s}{r} \] \text{same units}
\[ r = \text{radius} \]
\[ \theta = \text{central angle (rad)} \]
\[ \omega = \frac{\theta}{t} \]
\[ \omega = \text{angular speed} \]
\[ v = r \omega \]
\[ v = \text{linear speed} \]
\[ \text{at a point on rim} \]
\[ t = \text{time} \]

DEGREES, MINUTES AND SECONDS

\[ 1^\circ = 60' \]
\[ 1' = 60'' \]

POLAR COORDINATES
Polar \( \rightarrow \) Rectangular \((r, \theta) \rightarrow (x, y)\)

\[ x = r \cos \theta \]
\[ y = r \sin \theta \]

Rectangular \( \rightarrow \) Polar \((x, y) \rightarrow (r, \theta)\)

\[ r = \sqrt{x^2 + y^2} \]
\[ \theta = \cos^{-1} \left( \frac{x}{\sqrt{x^2 + y^2}} \right) = \frac{x}{r} \]
\[ \theta = \sin^{-1} \left( \frac{y}{\sqrt{x^2 + y^2}} \right) = \frac{y}{r} \]

VECTORS
if \( \vec{u} = (a, b) \) then: \( \vec{u} + \vec{v} = (a + c, b + d) \)
\[ \vec{v} = (c, d) \]
\[ ||\vec{u}|| = \sqrt{a^2 + b^2} \]

ANGLES OF ELEVATION AND DEPRESSION

SINE AND COSINE LAWS
Cosine Law (use with SAS or SSS)
\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

Sine Law (use with AAS, SSA)
\[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \]
[Note: the SSA case may yield 0, 1 or 2 solutions.]

ANGLE SPECIFICATION TECHNIQUES

Directed Angles
- used for trig class
- +ve angles: start at the positive x-axis and go CCW
- -ve angles: start at the positive x-axis and go CW
- angles higher than 360° or 2\( \pi \) rad are possible

Heading, Bearing or Course
- used by airplanes and boats
- start at North, go CW
- angle of arrival (coming into a destination) is 180° away from the angle leaving a location (e.g., if angle of departure is 30°, angle of arrival will be 210°)

Compass Directions
- two forms:
  1) “N 50° E” starts at North or South and moves CW or CCW eastward or westward
  2) “65° south of west” starts at second compass point and moves CW or CCW toward the first compass point
- in form (2), every direction can be expressed two different ways

Wind Direction
- used in flying, weather reports
- wind direction is the direction the wind is blowing \textit{from}, not \textit{to}, e.g., a west wind blows from west to east
- find wind direction using above techniques; draw the vector pointing 180° away from the indicated angle