Slope

The **slope** of a line is a numerical way of describing the “steepness” and the direction of the line. It tells you nothing about the position of the line—where it is on the xy-plane.

To find the slope of a line from its equation, rewrite the equation in the form $y = mx + b$, so that the $y$ is isolated on one side. The coefficient on $x$ is the slope ($m$).

**Example 1:** Find the slope of the equation $9x + 3y = 8$

**Solution:** Isolate the $y$, and then extract the $x$ coefficient:

$$
9x + 3y = 8 \\
3y = -9x + 8 \\
y = -3x + \frac{8}{3}
$$

Therefore the slope is $-3$.

To find the slope between two points, use the mnemonic “**rise over run**” to remember what to do. If the coordinates of the points are $(x_1, y_1)$ and $(x_2, y_2)$:

$$
m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

**Example 2:** Find the slope of the line containing the points $(3, 6)$ and $(7, 10)$.

**Solution:** Use the rise-over-run formula:

$$
m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 6}{7 - 3} = \frac{4}{4} = 1
$$

Therefore the slope is $1$.

**Vertical lines** have an infinite, or undefined, slope since there is no “run” to the line; they don’t have an $x$ component to divide by. Vertical lines have equations in the form “$x = a$”.

**Horizontal lines** have a slope of $0$ since there is no “rise” to the line; there’s no $y$ component to divide into. Horizontal lines have equations in the form “$y = b$”. 

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EXERCISES

A. Find the slope of the line:

1) \( y = 5x - 3 \)  
4) \( x = 7 \)

2) \( 2y = 3x + 4 \)  
5) \( y = 8 \)

3) \( 3y + 4x = 5 \)

B. Find the slope of the line passing through each pair of points:

1) \( (1, 2), (3, 4) \)  
4) \( (-1, -3), (-9, -8) \)

2) \( (-3, 5), (-9, 10) \)  
5) \( (5, 6), (3, 6) \)

3) \( (4, 8), (4, 10) \)

SOLUTIONS

A. (1) 5 (2) \( \frac{3}{2} \) (3) \( -\frac{4}{3} \) (4) undefined (5) 0

B. (1) 1 (2) \( -\frac{5}{6} \) (3) undefined (4) \( \frac{5}{6} \) (5) 0