Many algebra problems have a single solution. If we have \( x + 3 = 8 \), the only answer is 5, and we can simply write \( x = 5 \). Other problems have multiple solutions or a range of solutions. There are two main ways to report answers to a question like this: interval notation and set notation.

**SET NOTATION**

Set notation is useful especially when we have a small, finite number of solutions, rather than a range of solutions. Take the equation \( x^2 = 9 \). There are two answers: 3 and -3. The list of all possible solutions to a problem is called its solution set and we should write it as a set using roster notation: \{−3, 3\}. The curly brackets (brace brackets) indicate that the answer is a list and that -3 and 3 are the only two acceptable answers. A solution in roster notation can have more than two numbers in the brackets.

It’s also possible to write the solution set to a problem by describing the solutions rather than by listing all of them. If we were asked, “What quantities of money can be withdrawn from a typical ATM?” and the ATM only dispenses $20 bills, then the answers are 20, 40, 60, 80, and so on. We could write \{20, 40, 60, ...\} as a way of listing the answers, or we could use set-builder notation to say how to calculate the answers: \{x | x = 20k, k ∈ \mathbb{N}\}. We read this as: The solutions are x, where x is 20 times k, and k is a natural number. This is a very precise answer, and more precise than your teacher is likely to ask for.

**INTERVAL NOTATION**

Interval notation is used whenever the answers to a problem form one or more continuous ranges of the number line. This frequently happens in inequalities.

Take for example, \( x^2 < 9 \). After some thought, it should be obvious that any number between -3 and 3 (but not including either number) is a solution to the problem. We express this in interval notation by enclosing the numbers that are the endpoints of the solution in brackets. We use round brackets or parentheses when the interval does not include the endpoints, and square brackets when the interval does include the endpoints. Here, since the solution interval doesn’t include those numbers on the end, we write: \((-3, 3)\). If the question were \( x^2 ≤ 9 \), -3 and 3 would be valid solutions. We use square brackets to mark endpoints included in the solution: \([-3, 3]\). We can also use both bracket types in expressing a solution. For \( 4 < x ≤ 7 \), the interval runs from 4 to 7, and 4 is not a solution, but 7 is. We write: \((4, 7]\).

Sometimes there’s no endpoint. For the question \( x ≥ 12 \), there’s a lowest possible solution, but no highest possible solution. We use the infinity symbol to show a lack of an endpoint, and we must always use a round bracket with it; infinity isn’t a number, so it can’t be a solution. We can’t include it as part of a solution set. We write \([12, ∞)\). We use \(-∞\) for solutions with no lowest endpoint: \( x ≤ 12 \) is expressed as \((-∞, 12]\).
Sometimes there are two intervals in the solution. For \(x^2 \geq 9\), any number greater than 3 is a solution and so is any number less than \(-3\), including 3 and \(-3\). The solutions fall in two intervals that are separated by a gap. We indicate this by borrowing a symbol from set theory: the union symbol, \(\cup\). This symbol is often translated as “or”, meaning that any point in this interval or that interval is a valid solution. The solution to \(x^2 \geq 9\) is written \((-\infty, -3] \cup [3, \infty)\). The solution to \(x \neq -4\) would be \((-\infty, -4) \cup (-4, \infty)\), indicating that the “gap” between the two intervals is only as large as the number \(-4\) itself.

If any number is a solution to an equation or inequality, as in \(x^2 \geq 0\), then we write \(\mathbb{R}\) in set notation (“all real numbers”) or \((-\infty, \infty)\) in interval notation. If no number is a solution, as in \(x^2 = -5\), then we write \(\emptyset\) in either notation. This is the symbol for the null set, meaning the solution set is empty.

**EXERCISES**

A. Is 0 a solution to the problem whose solution sets are given below?

1) \([-2, 5]\)

2) \((-2, 5]\)

3) \({-2, 5}\)

4) \((-7, 0]\)

5) \({x \mid x = k + 0.5, \text{ where } k \text{ is an integer}}\)

6) \({-15, -14, -13, \ldots 10, 11, 12}\)

7) \((-\infty, -5] \cup (-1, 6)\)

8) \([-3, -1] \cup [8, 10]\)

B. Write the solution sets to the following problems in the appropriate notation.

1) \(x^2 < 25\)

2) \(|x| \leq 8\)

3) \(x < 7 \text{ and } x \geq 2\)

4) List the factors of 12.

5) \(x > 7 \text{ and } x \leq 2\)

6) How much money can be withdrawn from ATMs that dispense $20 and $50 bills?

7) \(x > 13\)

8) \(x < 7 \text{ or } x \geq 2\)

9) State the domain of \(\frac{x + 5}{x - 4}\).

10) \(x > 7 \text{ or } x \leq 2\)

**SOLUTIONS**

A: (1) yes (2) yes (3) no (4) yes (5) no (6) yes (7) yes (8) no

B: (1) \((-5, 5]\) (2) \([-8, 8]\) (3) \([2, 7]\) (4) \({1, 2, 3, 4, 6, 12}\) (5) \(\emptyset\) (6) \({20, 40, 50, 60, 70…}\) or \({x \mid x = 20a + 50b, \text{ where } a \text{ and } b \text{ are whole numbers; } a \text{ and } b \text{ can’t both be } 0}\) (7) \((13, \infty)\) (8) \((-\infty, \infty)\) or \(\mathbb{R}\) (9) \((-\infty, 4) \cup (4, \infty)\) (10) \((-\infty, 2] \cup (7, \infty)\)