**Significant Figures**

For any measurement, the number of significant figures (or “sig figs”) is equal to those digits that can be recorded accurately, plus one digit that must be estimated.

1. **All non-zero figures are significant.** The number 2.3163 has five sig figs.

2. **Zeroes between significant figures are significant.** The number 10.04 has four sig figs.

3. **Trailing zeroes after a decimal point are significant.** The number 2.40 has three sig figs.

4. **Zeroes to the left of the first non-zero digit are not significant.** The number 0.005075 has four sig figs, not eight. The number 0.63 has two sig figs, not three.

5. **Trailing zeroes in a whole number are not significant.** If any zeroes like this are significant, the number should be expressed in scientific notation to avoid any ambiguity. The number 2000 has one significant digit, but the number $2.00 \times 10^3$, which expresses the same quantity, has three significant digits. (In some books, they will write “2000” with a decimal point to show that the trailing zeroes are significant.)

6. **Leading zeroes in whole numbers are not significant.** An odometer reading of 001753 has four significant figures, not six.

7. **Exact numbers have infinite significant digits.** Numbers that are arrived at by counting rather than by measuring do not affect the number of significant digits in the answer. In statements like “The diameter is $2 \times r$”, “1 m = 100 cm”, or “15 oscillations of the pendulum”, the numbers 2, 100 and 15 have infinite sig figs.

8. **Use the best measurements you can find for “book” values.** Since inexact numbers affect the number of significant figures in a calculation, try to use the most accurate values you can. Instead of typing “3.14” for π, use the pi button of your calculator. When calculating molecular masses in Chemistry, two decimal places is usually enough. “H = 1.00794 amu” has six significant figures, but “1.01” is usually good enough.

**ADDITION AND SUBTRACTION**

9. **When adding and subtracting measured numbers, the answer should have as many decimal places as the number in the question with the fewest decimal places.**

   $2874.5 + 3.763 = 2878.263$. The number in the calculation with the fewest decimal places, 2874.5, has one, to the answer should be written with one decimal place: 2878.3. (Note that 3.763 has fewer sig figs, but since 2874.5 is less precise, it’s the measurement that limits how we can express our answer.)
MULTIPLICATION AND DIVISION

(10) When multiplying and dividing measured numbers, the answer should have as many sig figs as the number in the question with the fewest sig figs.

\[
\begin{array}{c}
\frac{2.300}{1.22} \times 2.3555 = 4.40696721 \\
\end{array}
\]

The number in the calculation with the fewest sig figs, 1.22, has three, so the answer should be written with three: 4.44

(11) When raising a measured number to a power, or taking the root of a measured number, the answer should have as many sig figs as the measurement does. The exponent is not considered for sig figs.

\[
\sqrt{29.1} = 5.394441584 \ldots
\]

The radicand has three sig figs, so we express the root to the same precision: 5.39. \(17.3^2 = 299.29 \approx 299\), with three sig figs, even though the exponent “2” is only a single digit number. It’s exact, as per Rule 7.

FOR LOGARITHMS

(12) The answer to a logarithm should contain as many decimal places as the number of sig figs in the number to be logged. In “log 100”, the number to be logged, 100, has one significant figure, so the answer should have one decimal place: 2.0. In “log₆ 216”, the number to be logged has three sig figs, so the answer should have three decimal places: 3.000.

(13) Likewise, the answer to an antilogarithm should contain as many sig figs as the number of decimal places in the number to be antilogged. According to the calculator, \(\text{antilog } 1.628 = 42.46195639\ldots\). Since “1.628” has three decimal places, the answer should be expressed with three sig figs, as “42.5”.

COMBINATIONS

(14) Determine how many figures/decimal places should be in the answer according to (11) and (12), and round only at the end of the problem.

\[
\frac{76.98 \times (842.09 - 64.842)}{12.984} + 592.1
\]

The subtraction problem in the numerator (777.248) should have two decimal places, and five sig figs (777.25). The value of the fraction (4608.175527\ldots) should have four sig figs, and therefore no decimal places. The solution to the overall calculation (5200.275527\ldots) should have no decimal places, and should be expressed in scientific notation: \(5.200 \times 10^3\).

ROUNDING

(15) When rounding if the next number is 4 or less, round down. If it’s 5 or more, round up. To three sig figs, 2.4139 becomes “2.41” because 3 is less than four. To three sig figs, 2.4193 becomes “2.42” because 9 is more than five.
MEASUREMENT
(16) When representing the result of a measurement, it is customary to include all digits determined by the measuring device plus the first doubtful digit.

In the diagram on the right, we can tell easily that the bar is between 2.6 and 2.7 cm. We might estimate that the next digit makes this object 2.67 cm in length. So this ruler can make measurements up to two decimal places.

EXERCISES
A. State the number of significant digits:
   1) 2.8
   2) 2008
   3) 5.90
   4) 26 000
   5) 0.005
   6) 0.01030

B. Round to three significant figures:
   1) 1.625
   2) 2.003
   3) 2.005
   4) 98.27
   5) 13 145
   6) 8.997

C. Express each answer to the correct number of significant figures:
   1) 16.4 + 0.422 = 16.822
   2) 248.355 − 22.7 = 225.655
   3) 2.38 × 4.7 = 11.186
   4) 0.0078 ÷ 4.65 = 0.001677419355...
   5) 1.6684 × 2.900 × 1.27 = 6.1447172
   6) 715.7 − 2.99 = 712.71
   7) (1.77 × 10^4)(2.2 × 10^-8) = 3.894 × 10^-4
   8) \( \frac{(2.33 × 10^3)(3.22 × 10^{-6})}{(1.22 × 10^2)(9.44 × 10^{-5})} = 6.531497023... \times 10^{-9} \)
   9) \( (7.83 × 10^{-3})^2 = 6.13089 × 10^{-5} \)
   10) \( \frac{7.883(8.9+1.1097)}{3.114-1.7335} + 5.78 = 62.93788852... \)

SOLUTIONS
A. (1) two  (2) four  (3) three  (4) two  (5) one  (6) four
B. (1) 1.63  (2) 2.00  (3) 2.01  (4) 98.3  (5) 13 100, or 1.31 × 10^4  (6) 9.00
C. (1) 16.8  (2) 225.7  (3) 11  (4) 1.7 × 10^-3  (5) 6.14  (6) 712.7  (7) 3.9 × 10^-4
   (8) 6.53 × 10^-9  (9) 6.13 × 10^-5
   (10) 62.9: The addition in the numerator gives 10.0 (one dec. pl., three sig. fig.), and the subtraction in the denominator gives 1.381 (three dec. pl., four sig. fig.) so the fraction gives three sig figs, and one decimal place. The final addition’s answer should therefore have one decimal place.