The Problem Solver's Toolkit 2

MIXTURE PROBLEMS

# of pounds × Cost per pound = Value

	# of pounds	Cost per pound	Value
Brand A			
Brand B			
Total			

SOLUTION PROBLEMS

Amt. solution × % solute = Amt. solute

	Amt of solution	% salt (decimal)	Amt of salt
Sol'n A			
Sol'n B			
Total			

If "pure salt" is added, that's 100% salt.
If "water" is added, that's 0% salt.

DIGIT PROBLEMS
Let t = tens digit in the first number.
Let u = units digit in the first number.

The value of the first number is $10t + u$, and the value of the number with the digits reversed is $10u + t$.

When the question says, "A number is," they are referring to the value of the number.

COST-SHARING PROBLEMS

	# of students	Cost per student	Total cost
Actual	x	$\frac{500}{x}$ (larger)	500
Possible	x + 2	$\frac{500}{x+2}$ (smaller)	500

Larger − Smaller = Diff. in shares

MOTION WITH AIRPLANES & BOATS
AIRPLANES
Rate against wind (headwind)
= air speed − wind speed = $p - w$
Rate with wind (tailwind)
= air speed + wind speed = $p + w$

BOATS
Rate against current (upstream)
= still water rate − c = $b - c$
Rate with current (downstream)
= still water rate + c = $b + c$

WORK PROBLEMS

Part done per hr. × Time = Part of job done overall

Let t = time working together

	Hrs. Alone	Part done per hr.	Hrs. worked	Part done overall
A	5	$\frac{1}{5}$	t	$\frac{t}{5}$
B	4	$\frac{1}{4}$	t	$\frac{t}{4}$
			Total job done:	1

The total job done always adds up to 1 job.

SPECIAL CASE

For problems where one person starts to work alone before the second person joins in:

Let t = time working together

	Hrs. Alone	Part done per hr.	Hrs. worked	Part done overall
A	5	$\frac{1}{5}$	t	$\frac{t}{5}$
B	4	$\frac{1}{4}$	t + 2	$\frac{(t+2)}{4}$
	B works alone 2 extra hours ↑			1

Authored by Gordon Wong

INTEREST PROBLEMS

Example: A total of $1200 is invested in two funds. One fund earns 3% interest and the other earns 5%. The total interest earned is $54. Find the amounts invested.

Solution 1: We can solve this using one variable.

Let x = the amount invested at 3%.
Then 1200 − x is the amount invested at 5%.

	Principal	Rate	Interest
At 3%	x	.03	.03x
At 5%	1200 − x	.05	.05(1200 − x)
Total	1200		54

The Principal column should add up, and so should the Interest column. We can use the Interest column to make an equation:

$$.03x + .05(1200 - x) = 54$$

Multiply by 100 to get rid of the decimals:

$$\begin{aligned} 3x + 5(1200 - x) &= 5400 \\ 3x + 6000 - 5x &= 5400 \\ -2x + 6000 &= 5400 \\ 6000 - 5400 &= 2x \\ 600 &= 2x \\ 300 &= x \end{aligned}$$

So $300 was invested at 3%, and the rest, 1200 − x = $900, was invested at 5%.

Solution 2: We can also solve this using a system of equations:

Let x = the amount invested at 3%.
Let y = the amount invested at 5%.

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	Principal	Rate	Interest
At 3%	x	.03	.03x
At 5%	y	.05	.05y
Total	1200		54

The Principal column should add up, and so should the Interest column. We can use these columns to make a system of equations:

$$\begin{aligned} x + y &= 1200 \quad (1) \\ .03x + .05y &= 54 \quad (2) \end{aligned}$$

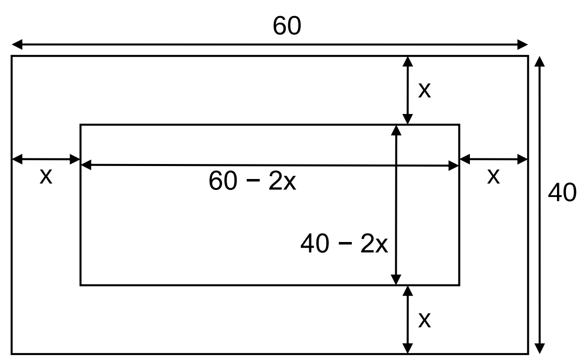
$$\begin{aligned} &(2) \times 100 \qquad & 3x + 5y &= 5400 \quad (3) \\ &(1) \times 3 \qquad & 3x + 3y &= 3600 \quad (4) \end{aligned}$$

From here, we can use either substitution or elimination to solve the system, and we get:

x = $300, y = $900

PICTURE FRAME

Example: A picture in its frame measures 60 cm × 40 cm. The painting has a border of fixed width on all sides. The total area of the border is 736 cm². How wide is the border?



Total area (Picture and frame): L × W
= (60 × 40)

Area of picture: l × w
= ([60 − 2x][40 − 2x])

Frame area = Total area − Area of picture