



The Problem Solver's Toolkit 1

TRANSLATIONS

ADDING	w <u>increased by</u> 4	$w + 4$
	2 <u>more than</u> k *	$k + 2$
	the <u>sum of</u> h and 8	$h + 8$
	the <u>total of</u> 6 and t	$6 + t$
SUBTRACTING	y <u>exceeds</u> x by 15 **	$y = x + 15$
	7 <u>decreased by</u> x	$7 - x$
	<u>difference between</u> k and 3	$k - 3$
	2 <u>less than</u> h *	$h - 2$
MULTIPLYING	y <u>exceeds</u> x by 15 **	$y - 15 = x$
	two-fifths <u>of</u> x	$\frac{2}{5}x$
	fifty percent <u>of</u> z	$50\% \cdot z$
	the <u>product of</u> 4 and n	$4n$
DIVIDING	8 <u>times</u> b	$8b$
	k <u>divided by</u> 7	$\frac{k}{7}$
	the <u>quotient of</u> c and 4	$\frac{c}{4}$
	the <u>ratio of</u> f to 6	$\frac{f}{6}$

* — “More than” and “less than” are written backwards from the order they appear in the question.

** — “Exceeds” can be written either way.

CONSECUTIVE INTEGERS

three <u>consecutive integers</u>	$x, x + 1, x + 2$
three <u>consecutive {even/odd} integers</u>	$x, x + 2, x + 4$
three <u>consecutive multiples of 4</u>	$x, x + 4, x + 8$

AGE PROBLEMS

For ages in the *future*, add years to the age now. For ages in the *past*, subtract years from the age now.

6 years ago	now	5 years from now
4	10	15
$x - 6$	x	$x + 5$

I am 10. In how many years... $10 + x$
 I am 23. How many years ago... $23 - x$

OPPOSITES

Two numbers are opposites... x and $-x$

COIN PROBLEMS

$$\begin{array}{|c|} \hline \text{Number} \\ \hline \text{of coins} \\ \hline \end{array} \times \begin{array}{|c|} \hline \text{Value} \\ \hline \text{of coin} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{Total value} \\ \hline \text{for that type} \\ \hline \text{of coin} \\ \hline \end{array}$$

Coin	Number	Value	Total
penny		1	
nickel		5	
dime		10	
quarter		25	

The “Value” and “Total” columns must be in the same unit (here, ϕ).

If the total of number of all coins is given, you may have to represent one type as “part of the whole”...

PARTS OF THE WHOLE

“A metal rod measuring 20 cm is cut in two pieces...” The lengths of the pieces can be written: $x, 20 - x$.

“A board 120 cm long is cut into three pieces. Two are the same length...” The three pieces can be written: $x, x, 120 - 2x$.

“Out of 18 coins, there are twice as many dimes as pennies, and the rest are nickels...” Pennies: x , dimes: $2x$, nickels: $18 - 3x$.



MOTION PROBLEMS

Think DiRT: Distance is Rate · Time.

Type of problem	Illustration	Key Relationships
One person travels, then a second traveller catches up after leaving from the same place at a later time.	<p>START</p>	$d_1 = d_2$ First traveller has time t . Second traveller has time $t - 3$.
A traveller changes transportation or speed at some point during a trip.	<p>START</p>	$d_{\text{total}} = d_1 + d_2$
Two travellers leave the same place at the same time going in opposite directions.	<p>START</p>	$d_{\text{total}} = d_1 + d_2$
A traveller goes to a destination, and then returns to his starting place.	<p>START</p>	$d_1 = d_2$

BUSINESS PROBLEMS	PERIMETER AND AREA PROBLEMS																
<p>COMMISSIONS $x\% \times \text{Total sales} = \text{Commission}$</p> <p>COST PROBLEMS $\# \text{ of items} \times \text{Cost/item} = \text{Total value}$</p> <p>INTEREST PROBLEMS $\text{Principal} \times \text{Rate} = \text{Interest}$</p> <p>MARKDOWNS $x\% \times \text{Original price} = \text{Markdown}$ $\text{Original price} - \text{Markdown} = \text{Sale price}$</p> <p>MARKUPS $x\% \times \text{Original price} = \text{Markup}$ $\text{Original price} + \text{Markup} = \text{Sale price}$</p> <p>PROFITS $\text{Revenue} - \text{Expenses} = \text{Profit}$</p> <p>SALES TAX $x\% \times \text{Marked Price} = \text{Sales Tax}$</p>	<table border="1"> <thead> <tr> <th></th> <th>Perimeter</th> <th>Area</th> </tr> </thead> <tbody> <tr> <td> <p>Square</p> </td> <td>$4x$</td> <td>x^2</td> </tr> <tr> <td> <p>Rectangle</p> </td> <td> $2L + 2W$ $= 2(L + W)$ </td> <td>$L \times W$</td> </tr> <tr> <td> <p>Triangle</p> </td> <td>$a + b + c$</td> <td>$\frac{1}{2}bh$</td> </tr> <tr> <td> <p>Circle</p> </td> <td>$2\pi r = \pi d$</td> <td>$\pi r^2 = \frac{\pi d^2}{4}$</td> </tr> </tbody> </table>		Perimeter	Area	<p>Square</p>	$4x$	x^2	<p>Rectangle</p>	$2L + 2W$ $= 2(L + W)$	$L \times W$	<p>Triangle</p>	$a + b + c$	$\frac{1}{2}bh$	<p>Circle</p>	$2\pi r = \pi d$	$\pi r^2 = \frac{\pi d^2}{4}$	
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<p>NUMBERS IN RATIOS</p> <p>Two numbers have a ratio of 4 to 3... $4x$ and $3x$</p> <p>Three numbers in the ratio 6:2:1... $6x$, $2x$ and x</p>	<p>ANGLE PROBLEMS</p> <p>complementary angles $x, 90 - x$ supplementary angles $x, 180 - x$</p> <p>angles in a triangle $a + b + c = 180$...in a right triangle $a + b = 90$...in an isosceles triangle $a + 2b = 180$</p>																

