Equivalent Fractions

There are many different fractions that have the same value. A score of $^{8}/_{16}$ on a test is the same as $^{20}/_{40}$ or $^{50}/_{100}$. These are called **equivalent fractions**.

CONVERTING A FRACTION

We can change an equation by doing the same thing to both sides. Similarly we can convert a fraction by multiplying the numerator and denominator of a fraction by the same number, or by dividing them by the same number. Sometimes this process is called **multiplying by 1**. You'll see why in these examples.

Example 1: Convert $^{3}/_{5}$ to an equivalent fraction by multiplying through by 6.

Solution:

$$\frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$$

Note: Since $^{6}/_{6}$ is equivalent to 1, just like any fraction of the form $^{x}/_{x}$, we are basically multiplying by 1.

Example 2: Write an equivalent fraction with the given denominator: $^{5}/_{6}$, 48.

Solution: What this question is asking is for a fraction, equivalent to $^{5}/_{6}$, but with a denominator of 48, or $^{x}/_{48} = {}^{5}/_{6}$.

We know we'll be multiplying the numerator and denominator of the fraction by the same number, but this time we don't know what the number is. We look at the denominators, since that's the part of the problem we know about. We have 6, and we want 48. What number do we multiply 6 by to get 48? We multiply by 8, so that's the number we use:

$$\frac{5}{6} \times \frac{8}{8} = \frac{40}{48}$$

The answer is $^{40}/_{48}$. This problem is good practice for adding or subtracting fractions.

We say that a fraction is in **lowest terms** when the numerator and denominator are both integers, and the integers have no common factors. It is useful to reduce a fraction to its lowest terms because it makes the fraction easier to understand. I could tell you that a math problem has an answer of $^{528}/_{1408}$, but it's hard to know what this answer means. If I reduce that horrible fraction, it becomes $^{3}/_{8}$, which is much easier to visualize.

Example 3: Reduce $^{24}/_{60}$ to lowest terms.

Solution: Look for common factors in the two parts of the fraction. When you find one, cancel it out and look again. If you don't see the greatest common factor right away, you may need to do this several times before you're done.

Authored by Darren Rigby

I see 4… $\frac{24}{60} \div \frac{4}{4} = \frac{6}{15}$

…and now I see 3. $\frac{6}{15} \div \frac{3}{3} = \frac{2}{5}$

Both those numbers are prime, so I'm done. $^{24}\!/_{60} = {}^{2}\!/_{5}$.

MIXED NUMERALS AND IMPROPER FRACTIONS

A fraction like $^{3}\!/_{5}$ is called a **proper fraction** because the larger number is in the denominator. A number like $^{5}\!/_{3}$ is called an **improper fraction** because the larger number is in the numerator, and so, the value of the fraction is greater than 1. It is better to write the answer in the form $1^{2}\!/_{3}$, which is a **mixed numeral**.

Example 4: Express $^{82}\!/_{12}$ as a mixed numeral, and reduce if necessary.

Solution: Divide the numerator by the denominator to get a quotient and a remainder. These answers will be the whole number part and the numerator of the mixed numeral.

82 ÷ 12 = 6 R 10, so $^{82}\!/_{12} = 6 + {}^{10}\!/_{12} = 6^{5}\!/_{6}$

To do this on your calculator, divide to get a decimal answer, subtract the whole part of the answer, and then multiply the remaining decimal by the divisor.

8 2 ÷ 1 2 =	6.83333333
− 6 =	0.83333333
× 1 2 =	10

We subtracted 6, so that's the whole number part, and the 10 is the numerator of the fraction.

Example 5: Express $7^{8}\!/_{9}$ as an improper fraction.

Solution: Multiply the denominator by the whole number part, and then add the numerator to get the new numerator. Keep the denominator.

$$7\overset{+}{\underset{\times}{}}\frac{8}{9} = \frac{7 \times 9 + 8}{9} = \frac{71}{9}$$

For this last type of question, it's better to reduce the fraction before you convert to a mixed number, so you have smaller numbers to work with.

EXERCISES

A. Convert to equivalent fractions by multiplying through by the given factor.

1) $\frac{4}{5}$, factor of 10

2) $\frac{2}{3}$, factor of 8

3) $\frac{7}{12}$, factor of 4

4) $\frac{2}{7}$, factor of 6

5) $\frac{15}{34}$, factor of 3

6) $\frac{42}{91}$, factor of $\frac{1}{7}$

B. Write equivalent fractions with the given denominators.

1) $\frac{3}{8}$, 24

2) $\frac{1}{6}$, 36

3) $\frac{3}{4}$, 48

4) $\frac{7}{8}$, 56

5) $\frac{4}{5}$, 30

6) $\frac{2}{9}$, 45

7) $\frac{1}{4}$, 100

8) $\frac{5}{12}$, 132

C. Reduce to lowest terms.

1) $\frac{3}{9}$

2) $\frac{50}{80}$

3) $\frac{6}{24}$

4) $\frac{16}{40}$

5) $\frac{28}{42}$

6) $\frac{200}{600}$

7) $\frac{125}{625}$

8) $\frac{77}{165}$

D. Express as mixed numerals and reduce if necessary.

1) $^{10}/_{3}$

2) $^{25}/_{4}$

3) $^{6}/_{4}$

4) $^{39}/_{7}$

5) $^{65}/_{2}$

6) $^{71}/_{6}$

7) $^{100}/_{14}$

8) $^{96}/_{8}$

E. Convert to improper fractions.

1) $1\frac{1}{3}$

2) $2\frac{2}{5}$

3) $3\frac{7}{12}$

4) $4\frac{4}{9}$

5) $5\frac{5}{18}$

6) $6\frac{1}{21}$

7) $7\frac{7}{25}$

8) $8\frac{15}{60}$

SOLUTIONS

A. (1) $^{40}/_{50}$ (2) $^{16}/_{24}$ (3) $^{28}/_{48}$ (4) $^{12}/_{42}$ (5) $^{45}/_{102}$ (6) $^{6}/_{13}$

B. (1) $^{9}/_{24}$ (2) $^{6}/_{36}$ (3) $^{36}/_{48}$ (4) $^{49}/_{56}$ (5) $^{24}/_{30}$ (6) $^{10}/_{45}$ (7) $^{25}/_{100}$ (8) $^{55}/_{132}$

C. (1) $^{1}/_{3}$ (2) $^{5}/_{8}$ (3) $^{1}/_{4}$ (4) $^{2}/_{5}$ (5) $^{2}/_{3}$ (6) $^{1}/_{3}$ (7) $^{1}/_{5}$ (8) $^{7}/_{15}$

D. (1) $3\frac{1}{3}$ (2) $6\frac{1}{4}$ (3) $1\frac{2}{4} \rightarrow 1\frac{1}{2}$ (4) $5\frac{4}{7}$ (5) $32\frac{1}{2}$ (6) $11\frac{5}{6}$ (7) $7\frac{2}{14} \rightarrow 7\frac{1}{7}$ (8) 12

E. (1) $^{4}/_{3}$ (2) $^{12}/_{5}$ (3) $^{43}/_{12}$ (4) $^{40}/_{9}$ (5) $^{95}/_{18}$ (6) $^{127}/_{21}$ (7) $^{182}/_{25}$ (8) $8\frac{1}{4} \rightarrow {}^{33}/_{4}$