Advanced Order of Operations

The standard **order of operations** tells us the priority of calculations in an expression, and makes writing expressions easier. You know BEDMAS: Brackets, Exponents, Division, Multiplication, Addition and Subtraction. There's more to order of operations than these six things.

B
E
DM
AS

First, there are not six levels to order of operations! There are four. When I teach people about order of operations, I tend to write it vertically, as shown on the right. Division and multiplication are really different expressions of the same operation. After all, $24 \div 3 = 24 \times \frac{1}{3}$. Any division problem can be rewritten as multiplying by the reciprocal of the divisor. The same can be said for addition and subtraction. In the same vein, $24 - 3 = 24 + (-3)$. When we have several division and multiplication steps to do in an expression, we do them in order from left to right; when we have several addition and subtraction steps to do in an expression, we do them in order from left to right.

It should also be said that no method of writing multiplication is more important than any other. The expressions 3×4, $3 \cdot 4$, and $3(4)$ are all identical. There are memes and viral challenges that circulate on social media that take advantage of people's incomplete understanding of order of operations to start arguments online. This is one example:

$$6 \div 2(1 + 2)$$

The "B" in BEDMAS only refers to resolving what's *inside* brackets, not what's in front of them. Then, $6 \div 2(1 + 2) = 6 \div 2(3) = 6 \div 2 \times 3 = 6 \times \frac{1}{2} \times 3 = 9$. The fact that "2(3)" includes a set of brackets doesn't mean it overrides the "DM" step in order of operations.

On the other hand, the "B" and "E" steps cover more than just (brackets) and just e^{xponents}. Roots of all kinds can also be expressed as exponents: $\sqrt{x} = x^{1/2}$, so they should be resolved as E in order of operations.

There are many instances of expressions that have the priority of "B" with there being any brackets around. These include:

- absolute value symbols — $|x - 7|$ cannot be simplified to $|x| - 7$; we would have to simplify "$x - 7$" first, just like we would for $(x - 7)^2$
- numerators and denominators of fractions — it should be obvious what's meant by $\frac{4 + 3}{5 - 2}$; especially when we enter these things into a calculator, the numerator and denominator should be put into brackets, to avoid getting $4 + \frac{3}{5} - 2$ instead. Radicands go here as well.
- named functions, such as sin x and log x — $\sin x + \pi$ is different from $\sin(x + \pi)$; use laws and identities to simplify if the argument of the function itself can't be simplified

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