VANCOUVER COMMUNITY COLLEGE

Inequalities

RULES

1. If you *multiply* or *divide* both sides of an inequality by a negative number, you must reverse the inequality sign. *Do not* reverse the sign when adding or subtracting negative numbers.
2. When working with absolute values in equations, isolate the absolute value on one side of the equation *before* proceeding.
3. $|a - b| = |b - a|$ (e.g. $|x - 3| = |3 - x|$)

INTERVAL NOTATION VS. SET-BUILDER NOTATION

Interval Notation:

1. Write the lowest number at the edge of the interval first.
2. If the number is included in the interval, use a [square bracket]. If the number is not included in the interval, use a (round bracket).
3. Do the same for the number at the other end of the interval.
4. If there is no smallest number (or no largest number) use $-\infty$ (or ∞). Since infinity is not a number, you must use (round brackets) on these symbols.
   Ex: "The numbers greater than or equal to 3" can be written as $[3, \infty)$
5. For more than one interval, connect them using the intersection symbol:
   $(-\infty, -1) \cup [3, \infty)$

Set-Builder Notation:

In curly brackets, write "{x |" and then write the inequality. Close the brackets. e.g. "The numbers greater than or equal to 3": $\{x \mid x \geq 3\}$. If your teacher is very mathematically precise, s/he may want you to say that the answer is a real number: $\{x \mid x \in \mathbb{R}, x \geq 3\}$

LINEAR INEQUALITIES

To solve a linear inequality:

1. Isolate the inequality for y.
2. Find the line for the graph as though it were an equation instead of an inequality.
   a. If the inequality symbol is "≤" or "≥", draw the line as a solid line.
   b. If the inequality symbol is "<" or ">", draw the line as a dotted line.
3. Select a point on the plane that is **not** on the line. Substitute the coordinates of the point into the inequality. (The origin, (0, 0), is a good choice because it's easy.)
   a. If the coordinates make the inequality true, shade the side of the line that has the point.
   b. If the coordinates make the inequality false, shade the other side.

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Authored by Gordon Wong

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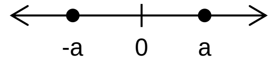
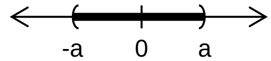
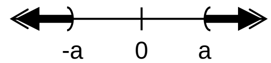
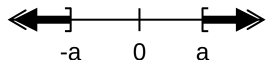
ABSOLUTE VALUES

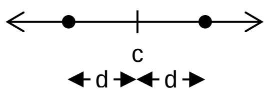
1. For any number a, a > 0:

TYPE	SOLUTION	GRAPH	SET OPERATION
$\lvert x\rvert = a$	$x = a$ OR $x = -a$		disjunction
$\lvert x\rvert < a$	$-a < x < a$		conjunction
$\lvert x\rvert > a$	$x < -a$ OR $x > a$		disjunction

We use a round mark on the number line to exclude a point, and a square mark to include a point. (Compare this with interval notation.) So the graph of $|x| \geq a$ would look like this:



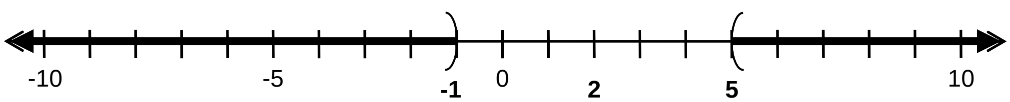
2. For any number d, d > 0, the graph of: $|x - c| = d$ is:



where c = the centre point, and
d = the distance from centre point to end point

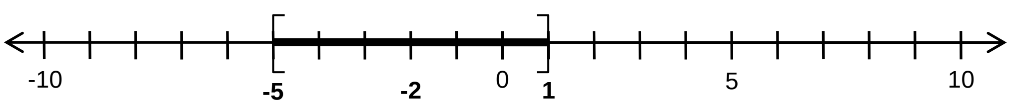
Example 1: Graph $|x - 2| > 3$.

Solution: The centre point is 2. The end points are 3 units away from the centre point. The graph consists of all x's such that the distance from 2 is greater than 3.



Example 2: Graph $|x + 2| \leq 3$.

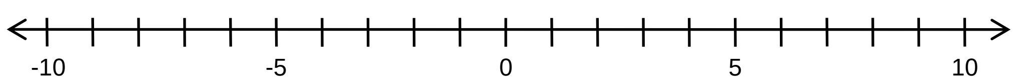
Solution: $c = -2$, since the expression must be written as $|x - (-2)| \leq 3$. The end points are 3 units away from the centre point. The graph consists of all x's such that the distance from −2 is less than or equal to 3.



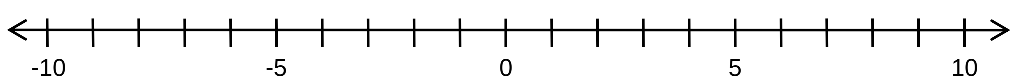
EXERCISES

A. Graph the following:

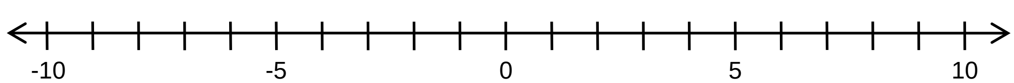
1) $\{x \mid x > 1\}$



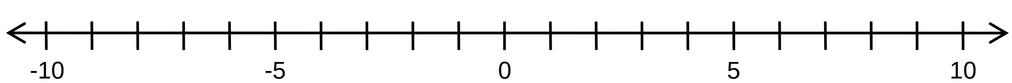
2) $(-\infty, 7]$



3) $(-\infty, -3) \cup [2, \infty)$



4) $x \leq -5$ OR $x > -2$



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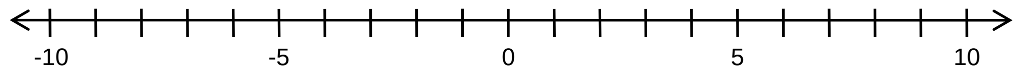
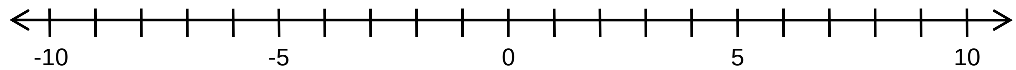
5) $x \ge 7$ AND $x \le 7$

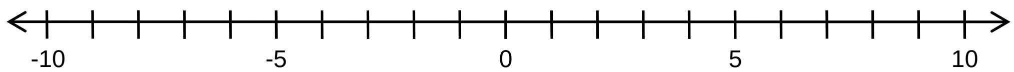


6) $x > 5$ AND $x \ge 6$

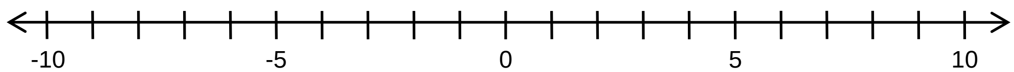


B. Solve, then graph:

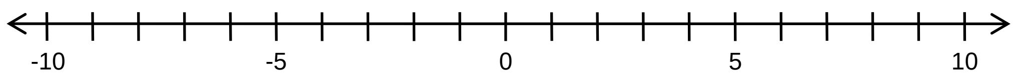
1) $-2 \le x + 1 < 3$



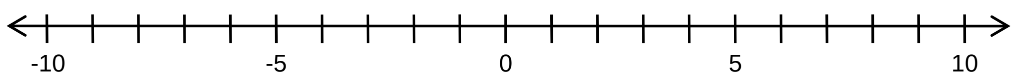
2) $-4 \le 4 - 2x < 4$



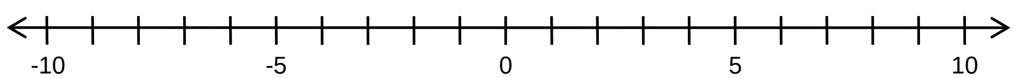
3) $9x \le -18$ OR $3(x - 2) > 0$



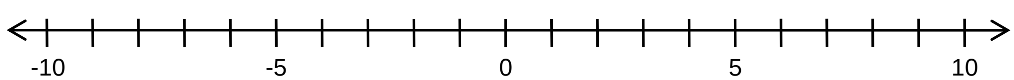
4) $|3x| = 9$



5) $|2g - 1| \ge 7$

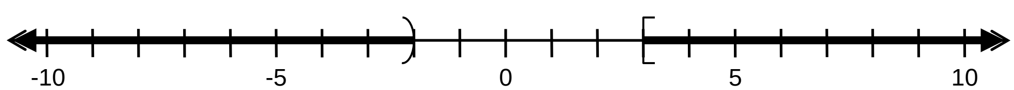


6) $1 + 2|x - 1| \ge 5$

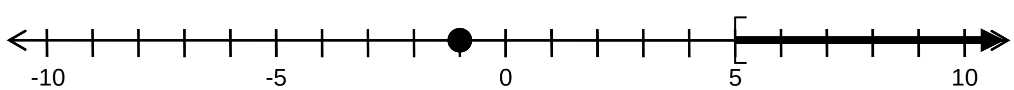


C. Express the following graphs in set-builder notation and in interval notation:

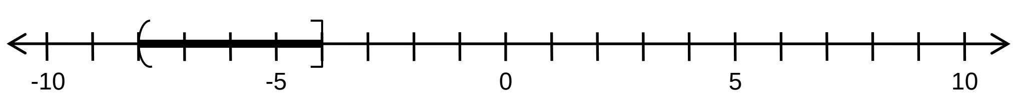
1)



2)



3)



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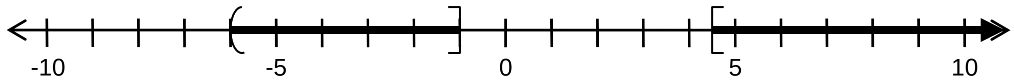
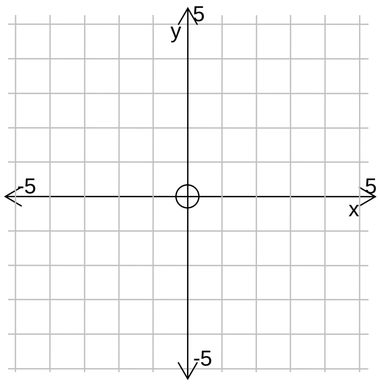
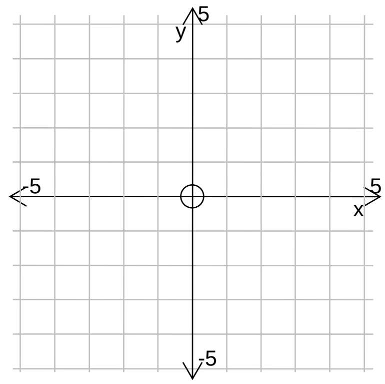
4) 

D. Graph the following linear inequalities:

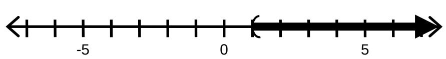
1) $y > 3x - 1$

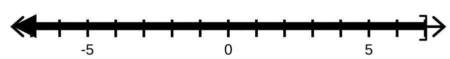


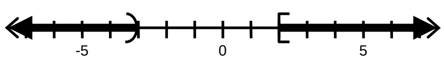
2) $y \le 2x + 1$

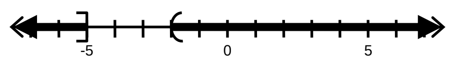


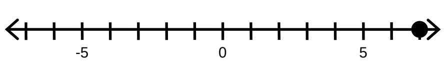
SOLUTIONS

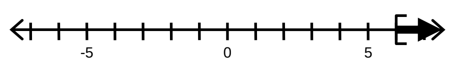
A. (1) 

(2) 

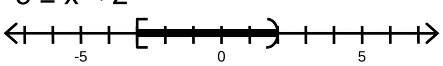
(3) 

(4) 

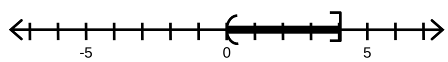
(5) 

(6) 

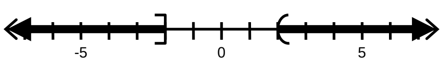
B. (1) $-3 \le x < 2$



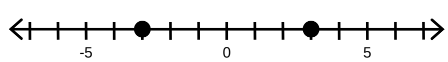
(2) $0 < x \le 4$



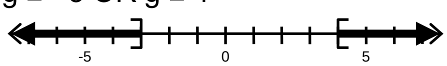
(3) $x \le -2$ OR $x > 2$



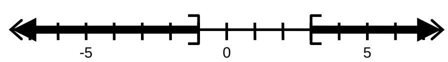
(4) $x = -3$ OR $x = 3$



(5) $g \le -3$ OR $g \ge 4$



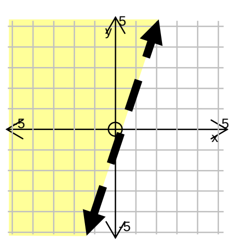
(6) $x \le -1$ OR $x \ge 3$



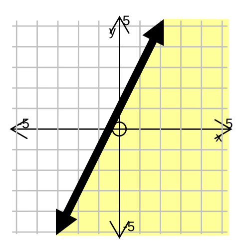
C. (1) $\{x \mid x < -2 \text{ OR } x \ge 3\}$; $(-\infty, -2) \cup [3, \infty)$ (2) $\{x \mid x = -1 \text{ OR } x \ge 5\}$; $[-1, -1] \cup [5, \infty)$

(3) $\{x \mid -8 < x \le -4\}$; $(-8, -4]$ (4) $\{x \mid -6 < x \le -1 \text{ OR } x \ge 4\tfrac{1}{2}\}$; $(-6, -1] \cup [4\tfrac{1}{2}, \infty)$

D. (1)



(2)



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