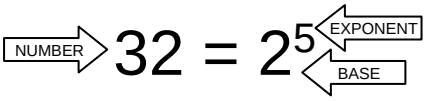
Exponents

An **exponent** is a number written above and to the right of an expression that indicates how many times to multiply that expression together. For example:

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$
$$= 32$$

The parts of an exponential expression have names.



There are **exponent laws** that describe how operations involving exponents should be carried out. They are listed below, along with examples of how they work:

$\mathbf{x^m \cdot x^n = x^{m+n}}$	$x^5 \cdot x^6 = x^{11}$; $x^5 \cdot x = x^5 \cdot x^1 = x^6$; $10^a \cdot 10^b = 10^{a+b}$
$\mathbf{\frac{x^m}{x^n} = x^{m-n}}$	$\frac{b^8}{b^2} = b^{8-2} = b^6$; $\frac{b^4}{b^6} = b^{4-6} = b^{-2} = \frac{1}{b^2}$
$\mathbf{(x^m)^n = x^{mn}}$	$(x^5)^2 = x^{10}$
$\mathbf{(xy)^m = x^m y^m}$	$(7a)^2 = 7^2a^2 = 49a^2$; $(mnop)^2 = m^2n^2o^2p^2$
$\mathbf{\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}}$	$(x/y)^2 = x^2/y^2$; $(2/3)^2 = 2^2/3^2 = 4/9$
$\mathbf{x^0 = 1}$	$5^0 = 1$; $(a^2b^7c^{12})^0 = 1$
$\mathbf{x^{-m} = 1/x^m}$	$x^{-2} = 1/x^2$; $3^{-2} = 1/3^2 = 1/9$
$\mathbf{1/x^{-m} = x^m}$	$\frac{1}{x^{-2}} = x^2$; $\frac{1}{4^{-2}} = 4^2 = 16$
$\mathbf{(x/y)^{-n} = (y/x)^n}$	$(2/3)^{-2} = (3/2)^2 = 3^2/2^2 = 9/4$
$\mathbf{\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}}$	$\frac{x^{-2}}{y^{-3}} = \frac{y^3}{x^2}$; $\frac{3^{-2}}{2^{-3}} = \frac{2^3}{3^2} = \frac{8}{9}$

NOTES:

1. To evaluate $-x^2$, square x first, *then* multiply the result by -1. So, $-4^2 = -16$. If a negative number is being squared, it needs to be put in brackets: $(-4)^2 = 16$.
2. While it is possible to simplify $x^a \cdot x^b$ by adding the exponents, the expression $x^a + x^b$ or $x^a - x^b$ cannot be simplified without evaluating x^a and x^b first.

Authored by Gordon Wong

EXERCISES

A. True or false? If the statement is false, evaluate the exponential expression:

1) $2^3 = 6$
2) $-2^2 = 4$
3) $-(-2)^2 = 4$
4) $3^{-2} \cdot 3^2 = 0$
5) $5^0 = 1$
6) $2 \cdot 2^2 = 16$
7) $1^{-2} = -\frac{1}{2}$
8) $5^{-1} = -5$
9) $0 \cdot 7^0 = 0$
10) $(2^3 + 3^2)^2 = 145$

B. Simplify and evaluate:

1) $(2x)^3$
2) $(-2)^4$
3) -2^4
4) $(3x)^0$
5) $3x^0$
6) $(-2x)^3$
7) $-(-2)^2$
8) $2 - 2^2$
9) $-2 - (-2)^2$
10) $-2 - (-2)^{-2}$

C. Multiply and simplify. Your final expression must not contain any negative exponents:

1) $5^{-2} \cdot 5^4$
2) $m^7 \cdot m^{-3}$
3) $(7x^3)(-2x^4)$
4) $(-3x^{-2})(-2x^{-3})$
5) $(5x^{-2}y^8)(-3x^{-3}y^{-5})$
6) $(4x^{-2}y^2)(2x^2y^{-2})$
7) $2x^4 \cdot 3x^2$
8) $f^{-3} \cdot f^3$
9) $5^6 \cdot 5^{-2} \cdot 5$
10) $(3x^4y^{-2}z^{-3})(-2x^2y^4z^{-6})$

D. Divide and simplify. Your final expression must not contain any negative exponents:

1) $\frac{5^9}{5^3}$

2) $\frac{5^5}{5^{-3}}$

3) $\frac{5^{-3}}{5^7}$

4) $\frac{x^{-2}}{x^{-4}}$

5) $\frac{x^6}{x^{-6}}$

6) $\frac{10h^2}{5h^3}$

7) $\frac{56x^2y^5}{-7x^2y^8}$

8) $\frac{x^3}{5x^3}$

9) $\frac{88x^{-2}y^{-2}}{11y^{-5}z^{-4}}$

10) $\frac{39xy^2z^0}{3x^2y^3z^{-4}}$

E. Simplify. Your final expression must not contain any negative exponents:

1) $(x^2)^2$

2) $(y^{-2})^6$

3) $(x^{-4})^{-2}$

4) $(3xyz)^2$

6) $(3x^2y^4)^{-2}$

7) $(3x^{-2}y^{-3})^2$

8) $(-2x^2y)^3$

9) $-(-2x^2)^3$

5) $(2x^{-2}y^6z^2)^3$

10) $(-4x^{-2}y^{-5})^{-2}$

F. Simplify. Your final expression must not contain any negative exponents:

1) $-2^4 + (-2)^4$

6) $\dfrac{4^{-2} + 2^{-4}}{32^{-1}}$

2) $(-x^2)(-x)^4$

7) $\dfrac{(-2)^4 - 2^2}{-2^2}$

3) $-(252x^2y^{-1}z^{-25})^0$

8) $-1^2 + 1^2 - (-1)^2 - 1$

4) $(ab^2)(-7a^2bc)(5c)$

9) $6^a \cdot 6^b \cdot 6^c$

5) $-2(2^4 - 3^2)^2$

10) $\left(\dfrac{2^{-2}x^{-2}}{x^3}\right)^{-2}\left(\dfrac{xy}{2^{-2}}\right)^{-3}$

SOLUTIONS

A. (1) False. 8 (2) False. −4 (3) False. −4 (4) False. 1 (5) True (6) False. 8
(7) False. 1 (8) True (9) True (10) False. $17^2 = 289$

B. (1) $8x^3$ (2) 16 (3) −16 (4) 1 (5) 3 (6) $-8x^3$ (7) −4 (8) −2 (9) −6 (10) $-\frac{9}{4}$

C. (1) $5^2 = 25$ (2) m^4 (3) $-14x^7$ (4) $6x^{-5} = \frac{6}{x^5}$ (5) $-15x^{-5}y^3 = -\frac{15y^3}{x^5}$ (6) $8x^0y^0 = 8$
(7) $6x^6$ (8) $f^0 = 1$ (9) $5^5 = 3125$ (10) $-6x^6y^{-2}z^{-9} = -\frac{6x^6}{y^2z^9}$

D. (1) $5^6 = 15{,}625$ (2) 5^8 (3) $5^{-10} = \frac{1}{5^{10}}$ (4) x^2 (5) x^{12} (6) $2h^{-1} = \frac{2}{h}$
(7) $-8x^0y^{-3} = -\frac{8}{y^3}$ (8) $\frac{1}{5}$ (9) $8x^{-2}y^3z^4 = \frac{8y^3z^4}{x^2}$ (10) $13x^{-1}y^{-1}z^4 = \frac{13z^4}{xy}$

E. (1) x^4 (2) $y^{-12} = \frac{1}{y^{12}}$ (3) x^8 (4) $9x^2y^2z^2$ (5) $8x^{-6}y^{18}z^6 = \frac{8y^{18}z^6}{x^6}$ (6) $\frac{1}{9}x^{-4}y^{-8} = \frac{1}{9x^4y^8}$
(7) $9x^{-4}y^{-6} = \frac{9}{x^4y^6}$ (8) $-8x^6y^3$ (9) $8x^6$ (10) $\frac{1}{16}x^4y^{10}$

F. (1) 0 (2) $(-x^2)(x^4) = -x^6$ (3) −1 (4) $-35a^3b^3c^2$ (5) $-(49) = -98$ (6) 4
(7) $\frac{16 - 4}{-4} = -3$ (8) $-1 + 1 - 1 - 1 = -2$ (9) 6^{a+b+c} (10) $\frac{x^7}{4y^3}$