Algebraic Expressions & Exponents

ALGEBRAIC EXPRESSIONS

Addition and Subtraction

Only *like terms* can be added or subtracted. *Like terms* have the same variable and exponent on the variable (e.g. 13, 5, and 2 are like terms; x, 3x, and 5x are like terms; $5x^2$ and $2x^2$ are like terms.) To combine like terms: add or subtract the *numerical coefficients* and keep the *variable part* the same. No numerical coefficient in front of the variable means a coefficient of 1.

3x + 3 + 4x + x…….. the three terms with x's are like terms
= (3 + 4 + 1)x + 3 …… add the *numerical* coefficients of the like terms
= 8x + 3

Simplification involving brackets. To remove the brackets, use the following rules:

- (+) sign or no sign in front of the brackets: drop the brackets and copy the terms inside the brackets with signs unchanged. (−7a + 5b − c) becomes −7a + 5b − c
- (−) sign in front of the brackets: drop the brackets and change the sign of every term inside the brackets. −(−7a + 5b − c) becomes 7a − 5b + c

Multiplication and Division

- To find the product of two or more single terms (monomials), find the product of the *numerical* coefficients and multiply by the product of their variables.
  5b (2b)
  = (5 × 2) (b × b) ……obtain the products of the *numerical* coefficients and the variables
  = 10 × b^2 = $10b^2$
- To find the product of a polynomial (multiple terms) and a monomial, multiply each term of the polynomial by the monomial.
  −2a (5a − 3b)
  = (−2a) (5a) − (−2a) (3b) …….multiply each term by (−2a)
  = $-10a^2$ + 6ab
- To find the product of two polynomials, multiply each term of the first polynomial by each term of the other polynomial and then simplify by combining like terms).
  (a + 2) (2a − 3) each term of the first polynomial is multiplied
  = a(2a − 3) + 2(2a − 3) ……. by the second polynomial
  = $2a^2$ − 3a + 4a − 6 …….. … carry out the multiplication
  = $2a^2$ + a − 6 …….. simplify
- To divide monomials, divide the numerical coefficients and the variables separately. Then multiply the answers.

  $12ab \div 6b = \left(\frac{12}{6}\right)\left(\frac{ab}{b}\right) = 2a$

- To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

  $(12a + 8) \div 4 = \frac{12a + 8}{4} = \frac{12a}{4} + \frac{8}{4} = 3a + 2$

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Substitution and Evaluation

To evaluate an algebraic expression means taking a given value for the variable and plugging it in (substituting in for that variable) to find the value of the function would be.

Evaluate $5x - 2y + 5$ for $x = 2$, $y = 1$

$= 5(2) - 2(1) + 5$…… replace x with 2 and y with 1

$= 10 - 2 + 5 = 13$

Practice Problems

Simplify:

1. $2a + 3c + 4a + 6c$
2. $3c - 2b + 3c^2 - 5b$
3. $d - 0.08d$
4. $(a + 3c) - (1.5a + 6c)$
5. $17m - 5m - m$
6. $(r^2 - 2rs + 3s^2) - (5r^2 + 6rs - 7s^2)$
7. $2d \cdot 4d$
8. $4(x + 3)$
9. $m(y - 7)$
10. $(5x - 4y)(x + 2y)$
11. $2(b - 2)(2b - 4) - 3(2b - 6)(b + 1)$
12. $15a \div 3a$
13. $20ab \div 4$
14. $(-27x^3) \div (-3x)$
15. $(32m - 24) \div 4$
16. $(15c^5 - 25c^3 - 10c^2) \div (-5c^2)$

Evaluate the following expressions (round answers to 2 decimal places):

1. $2ab + 3 - 2c$ for $a = 3$, $b = 2$, $c = 1$
2. $FV(1 - rt)$ for $FV = \$1500$, $r = 0.125$, $t = {}^{300}/_{365}$
3. $\frac{2NC}{P(n+1)}$ for $N = 42$, $C = 50$, $P = 1600$, $n = 0.025$
4. $\frac{I}{Pt}$ for $I = 120$, $P = 840$, $t = 0.75$
5. $\frac{FV}{1+rt}$ for $FV = \$1780$, $r = 0.095$, $t = {}^{241}/_{365}$

Solutions

Simplify:

1. $6a + 9c$
2. $3c^2 + 3c - 7b$
3. $0.92d$
4. $-0.5a - 3c$
5. $11m$
6. $-4r^2 - 8rs + 10s^2$
7. $8d^2$
8. $4x + 12$
9. $my - 7m$
10. $5x^2 + 6xy - 8y^2$
11. $-2b^2 - 4b + 34$
12. 5
13. $5ab$
14. $9x^2$
15. $8m - 6$
16. $-3c^3 + 5c + 2$

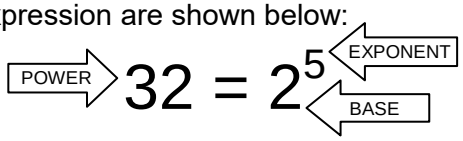
Evaluate: 1. 13 2. $1,345.89 3. 2.56 4. 0.19 5. $1,674.94

EXPONENTS

An **exponent** is a superscript number written to the right of a term or expression that indicates how many times to multiply that term or expression by itself. For example:

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$
$$(x + 3)^3 = (x + 3) \cdot (x + 3) \cdot (x + 3)$$

The parts of an exponential expression are shown below:



We read this equation as the power equals a number or base raised to the exponent.

The **exponent laws,** along with examples of how they work, are listed below:

1. $\mathbf{x^m \cdot x^n = x^{m+n}}$ $x^5 \cdot x^6 = x^{11}$; $10^a \cdot 10^b = 10^{a+b}$
2. $\mathbf{x^m \div x^n = x^{m-n}}$ $b^8 \div b^2 = b^{8-2} = b^6$
3. $\mathbf{(x^m)^n = x^{mn}}$ $(x^5)^2 = x^{10}$
4. $\mathbf{(xy)^m = x^m y^m}$ $(7a)^2 = 7^2a^2 = 49a^2$
5. $\mathbf{(x / y)^m = x^m / y^m}$ $(x / y)^2 = x^2 / y^2$
6. $\mathbf{x^0 = 1}$ $5^0 = 1$; $(a^2b^7c^{12})^0 = 1$
7. $\mathbf{x^{-m} = 1/x^m}$ $x^{-2} = 1/x^2$; $3^{-2} = 1/3^2 = \frac{1}{9}$
8. $\mathbf{1/x^{-m} = x^m}$ $1/x^{-2} = x^2$; $1/4^{-2} = 4^2 = 16$
9. $\mathbf{(x/y)^{-n} = (y/x)^n}$ $(2/3)^{-2} = (3/2)^2 = 3^2/2^2 = 9/4$
10. $\frac{\mathbf{x^{-m}}}{\mathbf{y^{-n}}} = \frac{\mathbf{y^n}}{\mathbf{x^m}}$ $\frac{x^{-2}}{y^{-3}} = \frac{y^3}{x^2}$; $\frac{3^{-2}}{2^{-3}} = \frac{2^3}{3^2} = \frac{8}{9}$

Notes:

a) $-2^4 \neq (-2)^4$ If the negative sign is inside the bracket it means (-2) raised to the fourth, or (-2)(-2)(-2)(-2) = 16. If the negative sign is outside, it means – (2^4) or –16.

b) If the base is a negative number with an even exponent, the power will be positive. If the base is a negative number with an odd exponent, the power will be negative. $(-3)^3 = -27$; $(-3)^2 = 9$

c) If the exponent is a FRACTION, we can express the exponent in radical form: $a^{1/3} = \sqrt[3]{a}$; $x^{2/5} = \sqrt[5]{x^2}$

d) The y^x button on your calculator allows you to evaluate powers. Enter the base, y^x and then the exponent to evaluate a power.

Practice Problems

Simplify:

1. $(2x)^3$
2. $(-3)^4$
3. $(3x)^0$
4. $(1/6)^{-2}$
5. $m^7 \cdot m^{-3}$
6. $1.204^{5/12}$
7. $(4 \cdot 5 \cdot 6x^3)^{1/3}$
8. $5^6 \cdot 5^{-2} \cdot 5$
9. $(3x^4 y^2 z^{-3})(-2x^2 y^{-4} z^{-6})$
10. $\dfrac{5^9}{5^3}$
11. $\dfrac{10h^2}{5h^3}$
12. $\dfrac{x^{-2}}{x^{-4}}$
13. $\left(\frac{1}{6}\right)^5 \div \left(\frac{1}{6}\right)^{-3}$
14. $\dfrac{39xy^2z^0}{3x^2y^3z^{-4}}$
15. $(-4x^2y^5)^{-2}$
16. $\left(\dfrac{2^{-2}x^{-2}}{x^3}\right)^{-2}\left(\dfrac{xy}{2^{-2}}\right)^{-3}$

Solutions

1. $8x^3$
2. 81
3. 1
4. 36
5. m^4
6. 1.08
7. $4.93x$
8. $5^5 = 3125$
9. $-6x^6y^{-2}z^{-9} = -\dfrac{6x^6}{y^2z^9}$
10. $5^6 = 15{,}625$
11. $2h^{-1} = 2/h$
12. x^2
13. $\left(\frac{1}{6}\right)^8$
14. $13x^{-1}y^{-1}z^4 = \dfrac{13z^4}{xy}$
15. $1/16x^4y^{10}$
16. $\dfrac{x^7}{4y^3}$