Order of Operations, Fractions & Percents

ORDER OF OPERATIONS

When finding the value of an expression, the operations must be carried out in a certain order. This order is known as the **order of operations**, and can be easily remembered using the acronym, BEDMAS. Note that division and multiplication are equal in the order performed — but they must be done from left to right in the expression. The same is true for addition and subtraction.

B	**B**rackets first, then
E	**E**xponents, then
D	**D**ivision and
M	**M**ultiplication, then
A	**A**ddition and
S	**S**ubtraction

Examples:

$3 + 3 \times 4$
$= 3 + 12$ [Multiplication first]
$= 15$ [Addition]

$(3 + 3) \times 4$
$= 6 \times 4$ [Brackets first]
$= 24$ [Multiplication]

$(17 - 4)^2 - 3 \times 2 + 4$
$= (13)^2 - 3 \times 2 + 4$ [Brackets first]
$= 169 - 3 \times 2 + 4$ [Exponents]
$= 169 - 6 + 4$ [Multiplication]
$= 167$ [Add and subtract from left to right]

Practice Problems

Solve the following problems:

1. $(3 + 4 \times 4 - 2) \times 2 + 3 \times 4$
2. $(3 + 4) \times [4 - (2 + 3 \times 2)] + 3 \times 4$
3. $-2 \times (3 + 4 \times 4 - 3^2) \times 2 + 3^2 \times 4$
4. $(3 + 8 \div 4 - 5) \times 2 + 3 \times 4$
5. $(3 + 4 \times 4 - 3) \times 2 - 3 + 4(3 \times 3^2)$
6. $22 - [(4 + 4 \times 4 - 3) \times 2 + 2] \div 4$
7. $(4 + 4) \times (4 - 4) + 2 + 3 \times (4 - 2^2)$
8. $48 + [2(16/4 + 3) + 2 - 5]^2 + 3$

Solutions

1. 46 2. −16 3. −4 4. 12 5.137 6. 13 7. 2 8. 172

FRACTIONS

In a fraction, the **numerator** is the number above the dividing line and the **denominator** is the number below. The denominator tells us how many pieces one whole object has been divided into, and the numerator tells us how many of those pieces the fraction represents. (Think of a test mark: how many right, out of how many questions.) A **proper fraction**'s numerator is smaller than the denominator. An **improper fraction** has a larger numerator than denominator. A **mixed number** contains both a whole number and a fraction.

Fractions can only be added or subtracted when they have the same denominator. To change the denominator of any fraction, multiply both the numerator and the denominator by the same number. For multiplication or division, fractions do not need a common denominator.

Example: $^{2}\!/_{9} + {}^{9}\!/_{18} = ?$
Solution: These fractions cannot be added together until they have a common denominator. The lowest common denominator is 18 (2 × 9 = 18). The first fraction is multiplied by $^{2}\!/_{2}$.

$$\frac{2\times2}{9\times2} = \frac{4}{18} \quad ; \quad \frac{4}{18} + \frac{9}{18} = \frac{13}{18}$$

Reducing a fraction to its lowest terms is done by dividing the numerator and denominator by a common number until there are no more common factors left to be divided out.

Example: Reduce $^{18}\!/_{27}$ to its lowest terms.
Solution: Both 18 and 27 have 3 as a common denominator; so divide 3 out. Both 6 and 9 also have 3 as a common denominator; divide 3 out again. 2 and 3 do not have any common factors so the fraction is in lowest terms.

$$\frac{18\div3}{27\div3} = \frac{6}{9} \quad ; \quad \frac{6\div3}{9\div3} = \frac{2}{3}$$

To **convert between improper fractions and mixed numbers**, use the following steps:

(1) Improper fraction to a mixed number :
   a. Divide the numerator by the denominator to get a quotient (whole number) and a remainder.
   b. The remainder goes over the original denominator and the fraction is reduced if possible.

Example: Express $^{82}\!/_{12}$ as a mixed number, and reduce if necessary.
Solution: Divide 82 by 12: 12 can go in 6 whole times (12 × 6 = 72), with 10 left over. The remainder goes over the original denominator.

82 ÷ 12 = 6 Remainder 10, so $^{82}\!/_{12} = 6 + {}^{10}\!/_{12} = 6^{5}\!/_{6}$

To do this on your calculator, divide to get a decimal. Subtract the whole number of the answer, and then multiply the remaining decimal by the divisor to find the numerator of the fraction.

8 2 ÷ 1 2 =	6.83333333
− 6 =	0.83333333
× 1 2 =	10

(2) Mixed number to improper fraction:
   a. Multiply the denominator by the whole number and add the number.
   b. Put the answer from part a over the original denominator.

Example: Express $7\frac{8}{9}$ as an improper fraction.
Solution:

$$7\overset{+8}{\underset{\times 9}{}}\frac{}{} = \frac{7 \times 9 + 8}{9} = \frac{71}{9}$$

To convert a fraction to a decimal, simply do the division. If the answer has a repeating decimal (doesn't terminate), make sure to indicate it by placing a dot or line above the repeating number.

Example: Express $\frac{7}{5}$ and $\frac{2}{3}$ in decimal form.
Solution: $\frac{7}{5} = 1.4$ $\frac{2}{3} = 0.6666\ldots = 0.\dot{6}$ or $0.\bar{6}$

To convert mixed numbers to decimal form divide the fraction part out and add to the whole number.

Example: Convert $6\frac{10}{11}$ to decimal form.

Solution: $6\frac{10}{11} = 6 + \frac{10}{11} = 6 + 0.9090\ldots = 6.\dot{9}\dot{0}$ *or* $6.\overline{90}$

Complex fractions contain one or more fractions in the numerator or denominator or both. As long as you follow order of operations properly, you can find the value of these expressions.

Example: $\dfrac{\$624}{1+0.15 \times \frac{80}{365}}$

Solution: First, deal with the denominator using BEDMAS. Multiply 0.15 by 80 and divide by 365. Add 1 to the result. Store this value in your calculator (STO 1) and divide $624 by the stored value (RCL 1). Always wait to round until the final answer.

$$\frac{\$624}{1 + 0.0329} = \frac{\$624}{1.0329} = \$604.14$$

Practice Problems

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1. Reduce each of these fractions to lowest terms.
   a. $\frac{7}{42}$ b. $\frac{15}{40}$ c. $\frac{34}{46}$ d. $\frac{105}{300}$ e. $\frac{93}{126}$ f. $\frac{112}{244}$ g. $\frac{345}{120}$ h. $\frac{260}{192}$

2. Find the value of the following additions/subtractions. Reduce the answer to lowest terms. If the answer is an improper fraction, convert it to a mixed number.
   a. $\frac{3}{5} + \frac{3}{4}$ b. $\frac{5}{8} - \frac{3}{7}$ c. $\frac{3}{4} + \frac{5}{12}$ d. $\frac{2}{3} - \frac{1}{5} + \frac{1}{6}$

3. Convert the following fractions into decimal form. Indicate repeating decimals where needed.
   a. $\frac{14}{8}$ b. $\frac{16}{24}$ c. $\frac{7}{3}$ d. $\frac{1}{6}$ e. $\frac{5}{11}$ f. $\frac{18}{16}$

4. Convert the following mixed numbers into decimal form. Indicate repeating decimals where needed.
   a. $3\frac{1}{9}$ b. $33\frac{2}{3}$ c. $5\frac{2}{5}$ d. $100\frac{7}{12}$ e. $9\frac{4}{25}$ f. $2\frac{7}{9}$ g. $6\frac{3}{8}$

5. Simplify each of the following (round to two decimal places).
   a. $\frac{\$74}{0.13\times\frac{200}{365}}$ b. $\frac{130}{3200\times\frac{118}{365}}$ c. $\frac{\$2578}{1-0.26\times\frac{270}{365}}$ d. $\frac{\$4839}{1+0.16\times\frac{84}{365}}$ e. $9016\left(1+0.14\times\frac{225}{365}\right)$

Solutions

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1. a. $\frac{1}{6}$ b. $\frac{3}{8}$ c. $\frac{17}{23}$ d. $\frac{7}{20}$ e. $\frac{31}{42}$ f. $\frac{28}{61}$ g. $\frac{23}{8}$ h. $\frac{65}{48}$

2. a. $\frac{27}{20} = 1\frac{7}{20}$ b. $\frac{11}{56}$ c. $\frac{7}{6} = 1\frac{1}{6}$ d. $\frac{19}{30}$

3. a. 1.75 b. $0.\overline{6}$ c. $2.\overline{3}$ d. $0.1\overline{6}$ e. $0.\overline{45}$ f. 1.125

4. a. $3.\overline{1}$ b. $33.\overline{6}$ c. 5.4 d. $100.58\overline{3}$ e. 9.16 f. $2.\overline{7}$ g. 6.375

5. a. \$1038.85 b. 0.13 c. \$3191.89 d. \$4667.15 e. 9794.09

PERCENTS (% means out of 100, divided by 100)

Basic percent rules:
- To convert a decimal to a percent, move the decimal 2 places to the RIGHT, or multiply by 100.
- To convert a percent to a decimal, move the decimal 2 places to the LEFT, or divide by 100.
- To convert a fraction to a percent, first convert the fraction into a decimal by dividing the denominator into the numerator. Then multiply by 100.
- To convert a percent to a fraction, divide by 100 and reduce. If the percent is a mixed number (whole number and fraction), convert to an improper fraction first, and then divide by 100.

Example: Convert $116\frac{1}{3}\%$ to a fraction in lowest terms.
*Solution: S*ince this is a mixed number, first convert to an improper fraction. Then divide by 100 and reduce:

$$116\frac{1}{3}\% = \frac{349}{3}\%$$

$$\frac{349}{3} \div 100 = \frac{349}{3} \div \frac{100}{1} = \frac{349}{3} \times \frac{1}{100} = \frac{349}{300}$$

There are three common types of percent problems:

1. Determine a percent of a given number.
   For example, what is 17% of 82?
   Method A is to use proportions. We must correctly determine if "82" is the "piece" or the "whole". Asking for a percent of a number tells us the number is the whole:

$$\frac{17}{100} = \frac{x}{82}$$

Use cross-multiplication: $\frac{17\times82}{100} = x$

$$13.97 = x$$

Method B:create an equation from the words and solve for the unknown. **Percents must always be converted to the fraction or decimal value in an equation.**

What is 17% of 82?
↓ ↓ ↓ ↓ ↓
x = 0.17 × 82
x = 13.94

2. Determine what percent one number is of another.
   For example, what percent of 48 is 16?
   Method A, use proportions:

$$\frac{16}{48} = \frac{x}{100}$$
$$x = \frac{1600}{48}$$
$$x = 33\frac{1}{3}\,\%$$

   Method B; translate to an equation and solve using algebra

What percent of 48 is 16?
↓ ↓ ↓ ↓ ↓
x × 48 = 16

$x = \frac{16}{48} = \frac{1}{3}$

(We still need to convert to a percent…)

$x = \frac{1}{3}\times 100 = \frac{100}{3}\% = 33\,\frac{1}{3}\%$

3. Determine a number when a percent of it is provided.
   For example, 3 is 16% of what?
   Method A, use proportions:

$$\frac{3}{x} = \frac{16}{100}$$
$$x = \frac{3\cdot 100}{16}$$
$$x = 18.75$$

Method B, translate to an equation and solve using algebra.

3 is 16% of what?
↓ ↓ ↓ ↓ ↓
3 = 0.16 × x
3 ÷ 0.16 = x
18.75 = x

Practice Problems

1. Change each of the following percents into a decimal.
   a) 0.15%
   b) 8.6%
   c) 124%
   d) $1\frac{1}{3}\%$
   e) $150\frac{3}{4}\%$
   f) $25\frac{2}{11}\%$

2. Change each of the following percents into a fraction in lowest terms. Convert any improper fractions to mixed numbers.
   a) 40%
   b) 0.35%
   c) 150%
   d) $60\frac{1}{2}\%$
   e) $110\frac{2}{3}\%$
   f) $20\frac{5}{8}\%$

3. Express each of the following as a percent:
   a) 0.059
   b) 0.278
   c) 1.15
   d) $\frac{4}{30}$
   e) $\frac{9}{8}$
   f) $\frac{4}{5}$

4. Find the requested quantity:
   a) What is 15% of 555?
   b) 25% of 44 is what?
   c) What percent of 50 is 23?
   d) 13 is what percent of 52?
   e) 120% of what is 42?
   f) 48 is 25% of what?
   g) 1.8 is what percent of 1.5?
   h) 52 is what percent of 13?

Solutions

1. a) 0.0015 b) 0.086 c) 1.24 d) $0.01\overline{3}$ e) 1.5075 f) $0.25\overline{18}$

2. a) $\frac{2}{5}$ b) $\frac{7}{2000}$ c) $1\frac{1}{2}$ d) $\frac{121}{200}$ e) $1\frac{8}{75}$ f) $\frac{33}{160}$

3. a) 5.9% b) 2.78% c) 115% d) $13.\overline{3}\%$ e) 112.5% f) 80%

4. a) 83.25 b) 11 c) 46% d) 25% e) 35 f) 192 g) 120% h) 400%